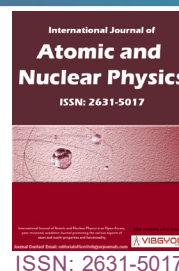


Study on Vibrational-Gamma Soft Properties in ^{124}Te Nucleus



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Abstract

We explained aspects of vibrational $-\gamma$ soft symmetry in ^{124}Te nucleus using different types of models such as Bohr-Mottelson Model (BM), Interacting Vector Boson Model (IVBM), Interacting Boson Model (IBM-1), and Doma-El-Gendy (D-G) relation. The factor $R = E_4^+ / E_2^+$ and energy level (E_2^+) of this nucleus have been studied and confirmed that ^{124}Te nucleus is vibrational to gamma soft aspects. A graph E_γ / J Vs. J of ^{124}Te nucleus has been investigated with the ordinary plots of rotational, gamma soft and vibrational limits. The known measured data of the yrast states of this nucleus are compared with four types of models BM, IVBM, IBM-1, and D-G. The theoretical calculations of these models are excellent with measured data.

Keywords

^{124}Te , BM, IBM-1, D-G, IVBM

Introduction

The study of tellurium isotopes is very important for nuclear physicist during the last two decades. The tellurium isotope with 72 numbers of neutrons is very much interesting to study the nuclear collectivity based on experimental and theoretical aspects. Because it is an even-even nucleus and easily produced E2 transitions where the low-lying

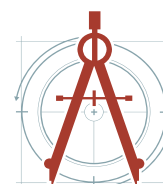
excitation levels are 2^+ , 4^+ , 6^+ and 8^+ states etc. This type of nucleus could be fruitfully described nuclear collective aspects using IBM-1 model [1]. According to his model there is no discrepancy between protons and neutrons and the collective states are defined by bosons number N_B . There are two bosons: d- ($L = 2$) and S- ($L = 0$) bosons. The collective states have three types of deformed symmetries: rotational SU(3), vibration U(5),

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and gamma-soft $O(6)$ [2]. Bohr-Mottelson found discrete revolving energy of a nucleus depends on the function of $J(J+1)$ [3]. Genev, et al. developed ground and octuplet band of nuclei by IVBM [4]. A novel technique for revolving energy subjected to J and Q was developed by Doma and El-Gendy [5].

We have studied the gamma soft character in ^{128}Ba [6,7]. The neutron-rich ^{128}Ba nucleus gives valuable information on the double nuclear shell closer to ^{132}Sn and raises the possibility to study the ^{124}Te nucleus. The ^{124}Te nucleus consists of 52 protons and 72 neutrons existed near to closed shell ^{132}Sn . The configuration of ^{124}Te nucleus is $\pi g_{9/2}^{+2}$ (proton particles) and $\nu h_{11/2}^{-0}$ (10 neutron holes). This configuration is excellent to study E2 gamma rays transition on the ^{124}Te nucleus by theoretically and experimentally. It gives valuable information on the yrast band for deformed nuclei.

Recently, the Doppler-shift attenuation method has been used to measure life time of ^{124}Te [8]. Ghita, et al. studied the low-lying states and critical symmetry in ^{124}Te nucleus by gamma ray spectroscopy [9]. The collective properties of vibrator- γ -soft energy of this nucleus were investigated by Ahmed using IBM-1 [10]. The shape coexistence in $^{118-128}\text{Te}$ was studied by Habri, et al. [11]. The B (E2) data of $\text{Pd}^{102-112}$ and Ru^{96-102} nuclei were examined [12-14]. Several vibrational-gamma soft properties IBM calculations have been reported that dealt with the IBM and IVBM models [15-19]. The aim of the present work is to interpret the collective behavior of yrast state of ^{124}Te nucleus by D-G, IBM-1, BM, and IVBM models. The nucleus with neutron number $N = 72$ should be transitional nuclei that are expected to play an important role in shape evolution as well as shape coexistence and are hence worth pursuing.

Materials and Methods

The details methodologies are explained in ref. [6]. For the three limits, Hamiltonian and eigen-values [20]:

U(5):

$$\left. \begin{aligned} \hat{H}_{U(5)} &= \varepsilon \hat{n}_d + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \\ E(n_d, \nu, L) &= \varepsilon n_d + K_1 n_d (n_d + 4) + K_4 \nu (\nu + 3) + K_5 L (L + 1) \end{aligned} \right\} \quad (1)$$

With

$$K_1 = 1/12 a_1$$

$$K_4 = -1/10 a_1 + 1/7 a_3 - 3/70 a_4$$

$$K_5 = -1/14 a_3 + 1/14 a_4$$

O(6):

$$\left. \begin{aligned} \hat{H}_{O(6)} &= a_0 \hat{P} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 \\ E(\sigma, \nu, L) &= K_3 [N(N+4) - \sigma(\sigma+4)] + K_4 \nu (\nu + 3) + K_5 L (L + 1) \end{aligned} \right\} \quad (2)$$

with

$$K_3 = 1/4 a_0$$

$$K_4 = 1/2 a_3$$

$$K_5 = a_1 - 1/10 a_3$$

SU(3):

$$\left. \begin{aligned} \hat{H}_{SU(3)} &= a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} \\ E(\lambda, \mu, L) &= K_2 [\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu) + K_5 L (L + 1)] \end{aligned} \right\} \quad (3)$$

with

$$K_2 = 1/2 a_2$$

$$K_5 = a_1 - 3/8 a_2$$

K_5, K_4, K_3, K_2, K_1 are strength parameters.

For the three limits, $R = E_\gamma / J$ and J are given [21,22]:

$$\text{Vibrator: } R = \frac{\hbar\omega}{J} \rightarrow 0 \text{ when } J \rightarrow \infty \tag{4}$$

$$\gamma\text{-soft: } = \frac{E2_1^+}{4}(1 + \dots) \rightarrow \frac{E2_1^+}{4} \text{ when } J \rightarrow \infty \tag{5}$$

$$\text{Rotor: } R = \frac{\hbar^2}{2\mathcal{G}}(4 - \frac{2}{J}) \rightarrow 4 \frac{\hbar^2}{2\mathcal{G}} \text{ when } J \rightarrow \infty \tag{6}$$

The details calculation of eigenvalues for U (5) - O (6) limits of yrast states [6,23], coefficient A, B, C in BM model [6,24], β, γ parameter of ground state band in IVBM [4,6] and the coefficient A, D, C in Doma - El-Gendy equation [5,6] depends on J and \mathcal{G} according to given in references.

Outcomes and Discussion

The first excited levels ($E2_1^+$) and the relations of $R = E4_1^+ / E2_1^+$ of ^{124}Te nucleus were explored vibration to gamma soft aspects are given in Table 1. The first excitation energy and $R = E4_1^+ / E2_1^+$ give the information of vibrator, gamma-soft and rotational symmetry of a nucleus.

Table 1 gives detailed evidence of the first excited levels, and the ratio of the first 4^+ to the first 2^+ states of ^{124}Te nucleus. The value of the first 2^+ and 4^+ level is 602.73 keV and 1247.65 KeV respectively. In Table 1 the ratio of $R = E4_1^+ / E2_1^+$ is 2.07 is signature of vibrational to gamma soft limit for the mention nucleus. The E-GOS curve E_γ / J Vs. J of the measured gamma energy/J [25] is drawn to vibrator, gamma-soft and rotator limit in Figure 1. It is shown that the measured value E_γ / J of yrast level in ^{124}Te of 2^+ levels rapidly decreases up to 6^+ level and then slowly increases up to 8^+ levels. The measured data E_γ / J is close to vibrator limit up to 6^+ and then near to gamma soft limit up to 8^+ levels. It clearly indicates that the yrast band of ^{124}Te nucleus is variable aspects.

Table 2 displays the value of $r(1 + 2/J)$ at $J = 2, 4$ and 6 are 0.054, -0.169 and 0.504 respectively and Figure 2 exhibits factor $r(1 + 2/J)$ of ^{124}Te are $0.169 \leq r \leq 0.504$ which are the vibration-gamma-soft property. The yrast bands of the ^{124}Te nucleus are calculated using BM, IVBM, IBM-1, and D-G methods. The best parameters and bosons numbers are given in Table 3. The boson number is 6. The values of $\hbar^2 / 2\mathcal{G}$ in ^{124}Te are 100.45; and do not satisfy the estimated parameters of this nucleus. Therefore this nucleus is not a purely rotational.

The experimental data [25] and calculated energies of the yrast level of ^{124}Te nucleus using different type of models are listed in Table 4. The calculated data are consistent with the reference of the ^{124}Te nucleus. The employed model reproduces the observed normal and intruder bands. Furthermore, the calculated energy spectra indicate that the mixing between the normal and intruder bands is apparent. In general, the approach is somewhat schematic and is considered on purely phenomenological grounds.

Table 1: The experimental data [25], of $E2_1^+$ and $R = (E4_1^+ / E2_1^+)$ of ^{124}Te nucleus.

Nucleus	^{124}Te
$E2_1^+$	602.73 keV
$R = (E4_1^+ / E2_1^+)$	2.07

Table 2: The $r(2/J + 1)$ Vs. J of ^{124}Te nucleus for yrast level.

The ratio $r(2/J + 1)$ Vs. J			
Nucleus	$J(2)$	$J(4)$	$J(6)$
^{124}Te	0.054	-0.169	0.504

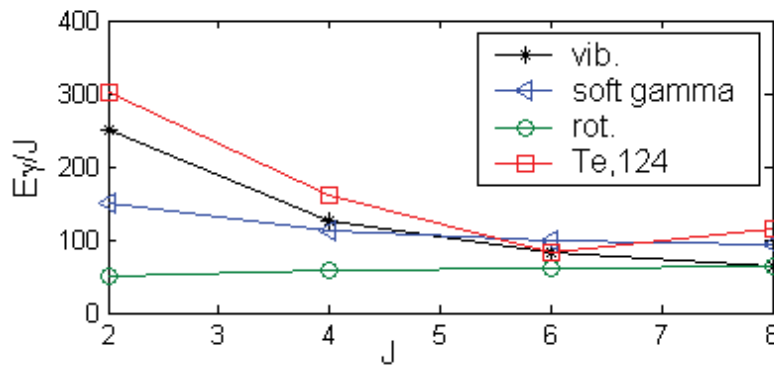


Figure 1: The E-GOS (E_γ / J Vs. J) plots of ^{124}Te related to the typical limit of rotator, vibrator and gamma soft.

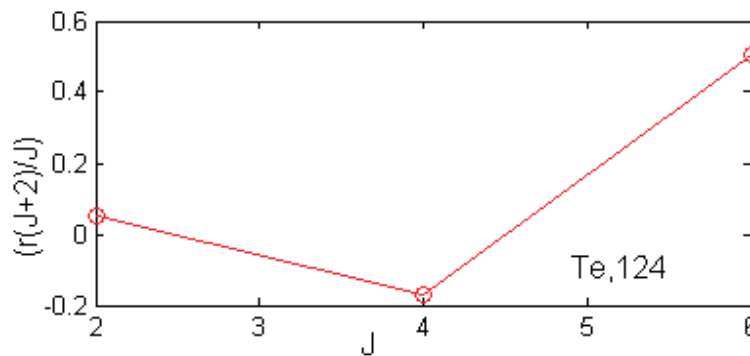


Figure 2: The $r(2/J + J)$ Vs. J of the yrast band.

Table 3: The boson number and parameters in (keV) of different type of models.

Nucleus	No. of bosons	IBM-1					IVBM	
		ϵ	K_1	K_2	K_4	K_5	β	γ
^{124}Te	6	515.49	-0.807	-0.01	35.38	-3.46	5.298	285.5
Nucleus		BM			DG			
		A	B	C	A	C	D	
^{124}Te		114.167	-2.388	0.0183	39.93	0.047	-0.001	

Table 4: Comparative data of experiment and theoretical models in yrasts state (keV).

J_1^+	^{124}Te				
	Ref.	Theory			
		IBM-1	IVBM	BM	DG
2	602.7	602.7	602.7	602.7	241.9
4	1248.6	1248.7	1247.8	1437.6	1249.7
6	1747.0	1936.3	1935.2	1736.2	1599.6
8	2665.1	2665.1	2665.1	2665.1	2780.9

Yet, such a study is still meaningful because a more realistic E2 transition approach would require this system. In addition the measured values of E2 transition rates of this nucleus were compared with the IBM-1 model [26]. We found the IBM-1 model for B(E2) is analytically deduced in U(5) limit for a few ground-state transitions in even ^{124}Te nucleus.

Conclusions

We studied the properties of the ^{124}Te nucleus using measured data and four different types of models: BM, IVBM, IBM-1, D-G. The first excitation of measured data is 602 keV and the ratio of

$E4_1^+ / E2_1^+$ is 2.07 in ^{124}Te nucleus indicate a vibration to gamma soft property. The yrast level of ^{124}Te nucleus in IBM-1, BM, IVBM, and D-G are consistent with the measured values. The factor $r(J + 2/J)$ of ^{124}Te at 2^+ , 4^+ and 6^+ is 0.054, -0.169 and 0.504 respectively. The low lying states of this nucleus are not pure rotator. The calculated energy spectra indicate that the mixing between the normal and intruder bands is apparent. This study confirmed vibrational- γ -soft aspects of the ^{124}Te nucleus. These results are comparable with the experimental results and extremely useful for compiling nuclear data table.

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