## Appendix A: Quantum-mechanical background of the formula (33) applied for the transition time *Δt*

We assume that for any time-dependent perturbation *ΔΗ(t)* of an originally unperturbed time-independent Hamiltonian *H0* does hold the Schroedinger equation:

 (1)

where

 (2)

In general *ψper* and *Εper* should depend on time *t*.

Let us differentiate the both sides of the first of equations present in (1) with respect to time. This gives



 (3)

where it has been taken into account in the second equation in (1) that the relation

 (4)

is satisfied. E0 in (4) is a time-independent eigenvalue due to the unperturbed Hamiltonian *H0*, but the perturbation energy *ΔΕ* may depend on time.

We assume that the dependence of *ΔΕ* on *t* is essential only at small *t* if the time *t=0* is considered as a beginning of the action of *ΔΗ(t)*. But at large t the *ψper* may become a stationary wave function, so the energy *Εper* in (4) ceases to be dependent on time leading to a constant *Εper*. In this case, because of (1), we obtain

 (5)

or

 (A5a)

The last two relations make the term *ΔΕ/Δt* in (3) negligible at large *t* *(Εper=Const)*.

For *Εper=Const* we obtain from (A5a) the equation for the harmonic oscillator having its solution equal to *ψper* which is periodic in time with frequency

 (6)

A well-known property of the harmonic *ψper* is that the average over *t* of any product

 (7)

Where *A* is an arbitrary constant term, should vanish. Evidently because of (A5a) the same vanishing property applies to the time average of the product

 (A7a)

Our aim is to apply the averaging process over time to the equation (3). This process eliminates the second time derivative entering the beginning of (3) giving the equation

 (8)

But because of a constant *A=E02* and the inferences on the formula (7), the equation (8) becomes reduced into

 (A8a)

from which

 (A8b)

Since *E0* is a constant, another averaging process over time applied to (A8b) eliminates the central term on its left-hand side. This gives the relation

 (9)

which differs from that obtained in (33) only by a constant multiplier before *ΔΕ.*

## Appendix B: Comparison of the present theory with quantum-mechanical results for the hydrogen atom

A typical application of *Δtn* is an expense rate of energy. With the aid of (33) we have

 (1)

Evidently in virtue of (32) the emission time rate of the energy *ΔΕn* is equal to

 (2)

This rate can be compared with quantum-mechanical results [15]. Since (2) is proportional to *(ΔΕn)2* in table 1 we present the factors entering *(ΔΕ)2* dependent on n, i.e. the expressions

 (3)

calculated according to (17), for the levels beginning with n = 1 up to n = 5.

The quantum-mechanical results for the total transition probabilities between the quantum states

 (4)

Where n = 1,2,3,4 and 5 and probabilities are expressed in 108 sec-1 units, are [15]:

 (5)

 (6)

 (7)

 (8)

 (9)

In table 2 the ratios of (3), calculated for different pairs of *n* and *n+1*, are compared with the ratios of the quantum-mechanical data obtained for the same pairs of *n* and *n+1* entering (5)-(9). A parallelism of the quantum-mechanical results with those calculated from the present semiclassical theory is evident.

**Table 1**: Expression (3) entering (Δ*Ε*n )2 in (2) calculated for different quantum levels *n*.

|  |  |
| --- | --- |
| ***n*** | **Expression (B3)** |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

**Table 2**: Ratios of expressions (B3) entering table 1 calculated for different pairs of the level indices *n* and *n+1* compared with the ratios of the transition intensity data given by the formulae (B5)-(B9).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***n*** | ***n+1*** | **Indices ratio** | **Ratios of expressions (B3)** | **Ratios of probabilities from the eqs. (B5)-(B9)** |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |