



Classical Dipole Emission Rate of Energy Compared with Quantum Dissipative Emission Rate

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Abstract

The time rate of energy emission in three quantum simple systems - the hydrogen atom, electron particle in a one-dimensional potential box and harmonic oscillator - has been calculated in two ways. The first one, based on the dipole approximation for the emitted energy, refers to classical electrodynamics. The other way is corresponding to quantum transitions of a single electron between definite energy levels. In this case the time of any transition is obtained by applying the Joule law characteristic for the dissipation of energy. The intensity of quantum transitions is found to be by several orders higher than the classical ones.

Introduction

According to classical electrodynamics any moving charged particle emits an amount of its energy during any time interval of its motion. A special interest is devoted usually to the periodic particle motions because such motions, presented for example by electrons, can be expected in the atoms as well as other quantum systems. Considering atoms - in their semiclassical picture - an electron is vibrating usually about the atomic nucleus. The frequency of vibration may not change in course of the motion, but the vibration amplitude necessarily decreases leading to a corresponding decrease of the oscillator energy. This means that the energy of a classical oscillator gradually decreases in course of a continuous vibrational process.

A different situation is provided, however, with the quantum atomic theory. If the Bohr picture characteristic for the hydrogen atom is assumed, the vibration, or - more precisely - a circulation of the electron, is going along the atomic orbits. For the motion on the orbit having the lowest energy this orbit can be incessantly preserved as an occupied object. On the other hand, in the case of the orbits corresponding to higher energies the motion on them can exist for some rather long time, but - at some moment which remains undefined for an observer - the position of the electron may be changed to the orbit of a lower energy. This effect is connected with a sudden emission of energy the amount of which is equal to the energy difference between the higher and lower orbit states. Certainly, if the end level is not a ground state, another emission process can take place afterwards at some unknown moment of time.

A classical rate of the emitted energy within a given interval of time, especially in the oscillator case, is a well-known quantity [1,2]. A different situation does exist, however, in case of the quantum electron transition called a "sudden" one above. In fact the quantum

theory of emission is of a statistical character and does not apply to a single electron transition. This is a common feature of both the old [3] and modern quantum theories [4-6]. The aim of the present paper is to cancel this statistical property. In fact the time of the transition of a single electron between two quantum levels can be calculated in a defined way on condition the transition energy is considered to be a Joule-Lenz dissipated energy known from the classical electrodynamics [7,8]. The formalism of that theory is outlined in section 2. In section 3 we compare the classical emission rate of simple quantum systems (hydrogen atom, a particle in the potential box and the harmonic oscillator) with the rate calculated with the aid of the formulae of section 2.

Classical Approach to the Emission Rate

The classical approach is based on the dipole approximation to the energy emission [1,2,9] by assuming that this kind of approach does represent the main part of the emitted energy. The oscillator equation is

$$m\ddot{u} + ku = 0, \quad (1)$$

where m is the mass of the oscillating particle and k is the elastic constant. The position vector \vec{u} is dependent on time:

$$\vec{u} = \vec{u}_0 e^{i\omega_0 t} \quad (1a)$$

and the circular oscillation frequency ω_0 is coupled to k and m by the formula

$$\omega_0^2 = \frac{k}{m}. \quad (2)$$

If the size of the electric charge present on the particle is e , with the vector \vec{u} is connected the dipole moment

$$\vec{p} = e\vec{u}. \quad (3)$$

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The emitted energy of the oscillator can be averaged over the time period

$$T = \frac{2\pi}{\omega_0} \tag{4}$$

In this case the emission rate of energy per unit of time is represented by the formula [1,9]

$$\eta = \frac{dE}{dt} = \frac{1}{3c^3} \dot{p}_0^2 = \frac{e^2}{3c^3} \ddot{u}_0^2 \omega_0^4 \tag{5}$$

where \ddot{u}_0 is the amplitude of \ddot{u} in (1a) and ω_0 is the emitted frequency.

In section 4 the above formalism is applied to calculation of the classical emission rate in three simple quantum systems because the electron moving in each of these systems can exhibit a typical oscillatory behaviour. In the next step of the mentioned Section the rate of the transition energy ΔE between two neighbouring quantum levels is calculated.

Quantum Approach to the Emission Rate

In fact this approach is based also on a classical formula in which the energy of transition ΔE between two quantum levels is an effect of the electric circuit produced in course of transition between these levels and the electric resistance associated with that circuit. A reference of ΔE to the time Δt necessary to obtain ΔE is assumed to be provided by the Joule-Lenz law for the dissipation effect of the energy ΔE .

The quantum character of such approach could be discovered only in a next step as a result of relation which does occur between ΔE and transition time of energy equal to Δt ; (29-31) below.

Moreover, since ΔE and Δt become fully defined quantities for a given transition, there exists no probabilistic background for the emission rate

$$\frac{\Delta E}{\Delta t} \tag{6}$$

of energy obtained in this way: both quantities entering the fraction (6) can be calculated in a strictly defined way. Such approach is leading to a fully non-probabilistic calculation of the rate of the energy emission - contrary to methods being at the basis of the emission calculations presented in [3-6].

As a beginning the Joule-Lenz formula for the emission of ΔE within the time interval Δt is applied:

$$P = Ri \tag{7}$$

Here R is the electric resistance of the circuit having intensity i associated with the emission. The R , i and the potential V of the emission are coupled by the standard formula

$$R = \frac{V}{i} \tag{8}$$

In the next step V is assumed to be connected with the transition energy ΔE by the relation

$$V = \frac{\Delta E}{e} \tag{9}$$

and the current intensity is represented by

$$i = \frac{e}{T} \tag{10}$$

where T is the period of time of the electron oscillation in a quantum system.

For particular systems considered in the present paper this gives

$$i_n = \frac{e}{T_n} = \frac{me^5}{2\pi n^3 \hbar^3} \tag{11}$$

for the hydrogen atom because the time period in this case is [10]

$$T_n = \frac{2\pi n^3 \hbar^3}{me^4} \tag{12}$$

For the electron particle in a one-dimensional potential box

$$i_n = \frac{e}{T_n} = \frac{ehn}{4mL^2} \tag{13}$$

because the particle energy E_n and its velocity v_n are coupled by the formula [7]

$$E_n = \frac{mv_n^2}{2} = \frac{n^2 \hbar^2}{8mL^2} \tag{14}$$

so

$$v_n = \frac{nh}{2mL} \tag{14a}$$

and the time period of the electron oscillation in the box having length L is

$$T_n = \frac{2L}{v_n} = \frac{4mL^2}{nh} \tag{14b}$$

In the case of the quantum harmonic oscillator its energy is

$$E_n = \left(n + \frac{1}{2} \right) \hbar \omega \approx n \hbar \omega \tag{15}$$

where the last step is valid for large n , and the oscillator frequency is

$$\omega = \omega_n = \frac{2\pi}{T} = \frac{2\pi}{T_n} \tag{15a}$$

valid for all quantum states n , so the time period of the oscillation

$$T = T_n = \frac{2\pi}{\omega} \tag{15b}$$

is the same for all states n giving

$$i = \frac{e}{T} = \frac{e\omega}{2\pi} \tag{16}$$

The ΔE needed to obtain V in (9) are [10]

$$\Delta E = -\frac{me^4}{2\hbar^2} \left[\frac{1}{(n+1)^2} - \frac{1}{n^2} \right] \approx \frac{me^4}{\hbar^2 n^3} \tag{17}$$

for the hydrogen atom on condition a transition between the quantum levels $n+1$ and n is considered.

For a similar pair of levels in a one-dimensional potential box we have

$$\Delta E = \frac{(n+1)^2 - n^2}{8mL^2} \hbar^2 \cong \frac{nh^2}{4mL^2} \tag{18}$$

(14) and for the quantum harmonic oscillator the separation between a pair of the neighbouring quantum levels of energy is the same for all pairs:

$$\Delta E = \hbar \omega. \tag{19}$$

This gives

$$R = \frac{V}{i} \cong \frac{me^4}{\hbar^2 n^3} \frac{1}{e} \frac{2\pi n^3 \hbar^3}{me^5} = \frac{2\pi \hbar}{e^2} = \frac{h}{e^2} \tag{20}$$

for the transition $n+1 \rightarrow n$ in the hydrogen atom on condition large n is considered,

$$R = \frac{V}{i} = \frac{nh^2}{4mL^2} \frac{1}{e} \frac{4mL^3}{ehn} = \frac{h}{e^2} \tag{21}$$

for a similar transition in a one-dimensional potential box, and

$$R = \frac{V}{i} = \frac{\hbar\omega 2\pi}{e e\omega} = \frac{h}{e^2} \quad (22)$$

for a transition done also between the neighbouring levels in the harmonic oscillator.

A characteristic point is that all R are the same (20-22) giving the result typical for the electric resistance in the integer quantum Hall effect [11].

In effect of a substitution of the data calculated in (11) and (20) into the formula (7) we obtain the following quantum emission rate

$$\frac{\Delta E}{\Delta t} = Ri_n^2 = \frac{h}{e^2} \left(\frac{me^5}{2\pi n^3 \hbar^3} \right)^2 = \frac{m^2 e^8}{2\pi n^6 \hbar^5} \quad (23)$$

for the hydrogen atom; for the electron particle in a one-dimensional potential box (7) gives

$$\frac{\Delta E}{\Delta t} = Ri_n^2 = \frac{h}{e^2} \left(\frac{ehn}{4mL^2} \right)^2 = \frac{h^3 n^2}{16m^2 L^4} \quad (24)$$

(13,21); finally in case of the quantum harmonic oscillator the data of (16) and (22) substituted to (7) give the emission rate

$$\frac{\Delta E}{\Delta t} = Ri^2 = \frac{h}{e^2} \frac{e^2 \omega^2}{(2\pi)^2} = \frac{\hbar \omega^2}{2\pi} \quad (25)$$

Another characteristic point than equal R values obtained in (20-22) concerns the products of ΔE and Δt . The Δt which can be readily obtained from (23-25) are:

$$\Delta t = \frac{\Delta E}{Ri^2} = \frac{me^4}{\hbar^2 n^3} \frac{2\pi n^6 \hbar^5}{m^2 e^8} = \frac{2\pi n^3 \hbar^3}{me^4} \quad (26)$$

for the transition examined in the case of the hydrogen atom,

$$\Delta t = \frac{\Delta E}{Ri^2} = \frac{nh^2}{4mL^2} \frac{16m^2 L^4}{h^3 n^2} = \frac{4mL^2}{nh} \quad (27)$$

for the transition in a one-dimensional potential box,

$$\Delta t = \frac{\Delta E}{Ri^2} = \hbar\omega \frac{2\pi}{\hbar\omega^2} = \frac{2\pi}{\omega} \quad (28)$$

for the transition considered in the harmonic oscillator.

The time intervals of (26-28) can be multiplied by the intervals ΔE which were at the basis of the mentioned results for Δt (17-19). In effect we obtain

$$\Delta E \Delta t = \frac{me^4}{\hbar^2 n^3} \cdot \frac{2\pi n^3 \hbar^3}{me^4} = 2\pi \hbar = h, \quad (29)$$

$$\Delta E \Delta t = \frac{nh^2}{4mL^2} \cdot \frac{4mL^2}{nh} = h, \quad (30)$$

and

$$\Delta E \Delta t = \hbar\omega \frac{2\pi}{\omega} = \hbar 2\pi = h, \quad (31)$$

respectively in the case of the hydrogen atom, a particle in the potential box and the harmonic oscillator.

Because of the results obtained in (29-31) the time rate of the quantum emission of energy takes the form

$$\eta^{\text{quant}} = \frac{\Delta E}{\Delta t} = \frac{(\Delta E)^2}{h} \quad (32)$$

in view of the fact that

$$(\Delta t)^{-1} = \frac{\Delta E}{h} \quad (33)$$

is given systematically by equations (29-31). In appendix A we show how (33) can be coupled with the quantum-mechanical calculations. In appendix B a comparison of the present theory with

quantum-mechanical calculations for the hydrogen atom has been done.

Comparison of the Classical and Quantum Rate of the Energy Emission

The classical emission rate depends both on the amplitude u_0 and the emitted frequency ω_0 (5):

$$\eta^{\text{class}} = \frac{e^2}{3c^3} u_0^2 \omega_0^4. \quad (34)$$

We assume the transition is going on between two neighbouring quantum levels, say $n+1$ and n . For the hydrogen atom, if n is large, the amplitudes of the both states are similar and can be assumed to be equal to the Bohr radii of the states [10], so

$$u_0 = r_n = \frac{n^2 \hbar^2}{me^2}. \quad (35)$$

For the electron particle in a one-dimensional potential box of length L we assume

$$u_0 = \frac{L}{2} \quad (36)$$

for all quantum states. For the harmonic oscillator being in state n the amplitude can be deduced from a classical relation between the energy and amplitude. Therefore for the energy (15) this gives [12]:

$$u_0 = \left(\frac{2E_n}{m\omega^2} \right)^{1/2} = \left(\frac{2n\hbar\omega}{m\omega^2} \right)^{1/2} = \left(\frac{2n\hbar}{m\omega} \right)^{1/2}. \quad (37)$$

The frequency ω_0 in all cases is assumed to that given by the transition energy

$$\Delta E \cong \hbar\omega \quad (38)$$

according to the formula

$$\omega_0 = \omega. \quad (39)$$

For the hydrogen atom the ΔE is assumed to be equal to the value given at the last step of (17). Together with the amplitude (35,39) we obtain for the classical emission rate the expression

$$\begin{aligned} \eta^{\text{class}} &= \frac{1}{3} \left(\frac{en^2 \hbar^2}{me^2} \right)^2 \frac{1}{c^3} \left(\frac{me^4}{\hbar^2 n^3 \hbar} \right)^4 = \frac{1}{3n^8} \frac{1}{\hbar^8} \frac{m^2}{c^3} e^{14} \\ &\cong \frac{1}{3} \frac{1}{n^8} \frac{1}{\hbar^8} \frac{m^2}{c^3} e^{14} \approx \frac{1}{3n^8} \frac{(4.8 \times 10^{-10})^{14}}{(10^{-27})^6 (3 \times 10^{10})^3} \\ &\approx \frac{1}{n^8} \frac{(4.8)^{14}}{3 \times 27} 10^{-8} \approx \frac{0.4 \text{ erg}}{n^8 \text{ sec}}. \end{aligned} \quad (40)$$

In course of the numerical calculations an approximate equality of the mass m and the Planck constant h is taken into account.

In the case of the electron oscillation in the potential box the energy interval for the transition is that given in (18). In the practical calculations let us assume that

$$L = 10^{-8} \text{ cm}. \quad (41)$$

With the aid of the amplitude (36), L given in (41) and formula (39) the classical emission rate is

$$\eta^{\text{class}} = \frac{1}{3} \left(\frac{eL}{2} \right)^2 \frac{1}{c^3} \left(\frac{nh^2}{4mL^2 \hbar} \right)^4 = \frac{n^4 (2\pi)^8 \hbar^4 e^2}{L^6 3 \times 4^5 m^4 c^3}$$

$$\cong \frac{n^4 (2\pi)^8 (4.8 \times 10^{-10})^2}{L^6 3 \times 4^5 (3 \times 10^{10})^3} = n^4 6.7 \frac{\text{erg}}{\text{sec}}. \quad (42)$$

Finally for the classical emission rate of the harmonic oscillator having frequency ω we obtain with the aid of (37) and relation

$$\omega_0 = \omega \quad (43)$$

the formula

$$\eta^{\text{class}} = \frac{1}{3} e^2 \frac{2n\hbar \omega^3}{m c^3} = \frac{2n\hbar (4.8 \times 10^{-10})^2}{3m (3 \times 10^{10})^3} \omega^3 \cong n 0.6 \times 10^{-5} \frac{\text{erg}}{\text{sec}}. \quad (44)$$

In the calculation presented in (44) we assumed that

$$\omega = 10^{15} \text{sec}^{-1}. \quad (45)$$

The quantum formulae for the emission rate are more simple since they depend solely on ΔE (31). For the hydrogen atom we have from the last step of (17) taken for ΔE

$$\eta^{\text{quant}} = \left(\frac{me^4}{\hbar^2 n^3} \right)^2 \frac{1}{h} = \frac{m^2 e^8}{\hbar^5 n^6 2\pi} \cong \frac{1}{n^6} \frac{(4.8 \times 10^{-10})^8}{(10^{-27})^3}$$

$$= 450 \times 10^3 \frac{1 \text{ erg}}{n^6 \text{ sec}}. \quad (46)$$

For the electron particle in the potential box the formula (31) combined with ΔE at the end of (18) gives

$$\eta^{\text{quant}} = \left(\frac{n\hbar^2}{4mL^2} \right)^2 \frac{1}{h} = \frac{n^2 \hbar^3 (2\pi)^3}{16m^2 L^4} \cong \frac{n^2 8\pi^3}{L^4 16} 10^{-27}$$

$$\approx n^2 \times 1.6 \times 10^6 \frac{\text{erg}}{\text{sec}} \quad (47)$$

on condition L equal to that given in (41) is applied.

The quantum emission rate (31) of the harmonic oscillator becomes

$$\eta^{\text{quant}} = \frac{(\hbar\omega)^2}{h} = \frac{\hbar\omega^2}{2\pi} \cong \frac{10^{-27} (10^{15})^2}{2\pi} \approx 0.16 \times 10^3 \frac{\text{erg}}{\text{sec}}. \quad (48)$$

By comparing (40,42,44) respectively with (46-48) we find that the quantum emission rate is in general by many orders larger than the classical rate. The ratio of η^{quant} to η^{class} indicates numerically how much a quantum transition is more "sudden" than a classical one.

Discussion and Summary

In the discussion two points merit to be mentioned here. The first one concerns the time intervals Δt calculated in (26-28). These intervals are coincident respectively with the time periods of the Bohr quantum states in the hydrogen atom (12) [10], the period of the particle motion in a one-dimensional potential box [7] and the period of the harmonic oscillator.

An interesting result is the ratio of the classical and quantum emission rate of the harmonic oscillator. This is given by the formula

$$\frac{\eta^{\text{class}}}{\eta^{\text{quant}}} = \frac{2n e^2 \hbar \omega^3}{3 m c^3} : \frac{\hbar\omega^2}{2\pi} = \frac{4\pi}{3} n \frac{e^2 \omega}{mc^3}. \quad (49)$$

The ratio (49) differs solely by the factor of

$$n \frac{2\pi}{\omega} = \frac{n}{\nu} = nT \quad (50)$$

from the damping constant

$$\gamma = \frac{2}{3} \frac{e^2}{mc^3} \omega^2 \quad (51)$$

of the classical emission [1,13,14]. The T entering (50) is the oscillation time period of the electromagnetic wave having the frequency ν .

A general attempt of the paper was to compare the emission rate between two neighbouring states of small quantum systems with the classical emission rate: the formalism shows that the quantum rate is many times larger than the classical one. In course of calculations the transition energy ΔE between quantum levels is assumed to represent a dissipative energy of the Joule character. The other results obtained for transitions are:

1. The electric resistance R characteristic for transitions is equal to the same constant number present in different systems, moreover this R is independent of the quantum index n ,

2. The size of the product of ΔE and the time interval Δt of transition attains the Planck constant h .

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Appendix A

Quantum-mechanical background of the formula (33) applied for the transition time Δt

We assume that for any time-dependent perturbation $\Delta H(t)$ of an originally unperturbed time-independent Hamiltonian H_0 does hold the Schroedinger equation:

$$i\hbar \frac{d\psi^{\text{per}}}{dt} = H^{\text{per}}(t)\psi^{\text{per}} = E^{\text{per}}\psi^{\text{per}} \quad (1)$$

where

$$H^{\text{per}}(t) = H_0 + \Delta H(t) \quad (2)$$

In general ψ^{per} and E^{per} should depend on time t .

Let us differentiate the both sides of the first of equations present in (1) with respect to time. This gives

$$i\hbar \frac{d^2\psi^{\text{per}}}{dt^2} = \frac{dH^{\text{per}}(t)}{dt}\psi^{\text{per}} + H^{\text{per}}(t)\frac{d\psi^{\text{per}}}{dt} \approx \frac{\Delta H(t)}{\Delta t}\psi^{\text{per}} + H^{\text{per}}(t)\frac{E^{\text{per}}}{i\hbar}\psi^{\text{per}} = \frac{\Delta E}{\Delta t}\psi^{\text{per}} + \frac{(E_0 + \Delta E)^2}{i\hbar}\psi^{\text{per}} \quad (3)$$

where it has been taken into account in the second equation in (1) that the relation

$$H^{\text{per}}(t) = E^{\text{per}} = E_0 + \Delta E \quad (4)$$

is satisfied. E_0 in (4) is a time-independent eigenvalue due to the unperturbed Hamiltonian H_0 , but the perturbation energy ΔE may depend on time.

We assume that the dependence of ΔE on t is essential only at small t if the time $t=0$ is considered as a beginning of the action of $\Delta H(t)$. But at large t the ψ^{per} may become a stationary wave function, so the energy E^{per} in (4) ceases to be dependent on time leading to a constant E^{per} . In this case, because of (1), we obtain

$$i\hbar \frac{d^2\psi^{\text{per}}}{dt^2} = E^{\text{per}} \frac{d\psi^{\text{per}}}{dt} = (E^{\text{per}})^2 \frac{1}{i\hbar} \psi^{\text{per}} \quad (5)$$

or

$$-\hbar^2 \frac{d^2\psi^{\text{per}}}{dt^2} = (E^{\text{per}})^2 \psi^{\text{per}}. \quad (A5a)$$

The last two relations make the term $\Delta E/\Delta t$ in (3) negligible at large t ($E^{\text{per}}=\text{Const}$).

For $E^{\text{per}}=\text{Const}$ we obtain from (A5a) the equation for the harmonic oscillator having its solution equal to ψ^{per} which is periodic in time with frequency

$$\omega^{\text{per}} = \frac{E^{\text{per}}}{\hbar}. \quad (6)$$

A well-known property of the harmonic ψ^{per} is that the average over t of any product

$$A\psi^{\text{per}}, \quad (7)$$

Where A is an arbitrary constant term, should vanish. Evidently because of (A5a) the same vanishing property applies to the time average of the product

$$A \frac{d^2\psi^{\text{per}}}{dt^2}. \quad (A7a)$$

Our aim is to apply the averaging process over time to the equation (3). This process eliminates the second time derivative entering the beginning of (3) giving the equation

$$\frac{\Delta E}{\Delta t}\psi^{\text{per}} + \frac{E_0^2 + 2E_0\Delta E + (\Delta E)^2}{i\hbar}\psi^{\text{per}} = 0 \quad (8)$$

But because of a constant $A=E_0^2$ and the inferences on the formula (7), the equation (8) becomes reduced into

$$\frac{\Delta E}{\Delta t}\psi^{\text{per}} + \frac{2E_0\Delta E}{i\hbar}\psi^{\text{per}} + \frac{(\Delta E)^2}{i\hbar}\psi^{\text{per}} = 0 \quad (A8a)$$

from which

$$\frac{1}{\Delta t} \psi^{\text{per}} + \frac{2E_0}{i\hbar} \psi^{\text{per}} + \frac{\Delta E}{i\hbar} \psi^{\text{per}} = 0. \tag{A8b}$$

Since E_0 is a constant, another averaging process over time applied to (A8b) eliminates the central term on its left-hand side. This gives the relation

$$\frac{1}{\Delta t} + \frac{\Delta E}{i\hbar} = 0 \tag{9}$$

which differs from that obtained in (33) only by a constant multiplier before ΔE .

Appendix B

Comparison of the present theory with quantum-mechanical results for the hydrogen atom

A typical application of Δt_n is an expense rate of energy. With the aid of (33) we have

$$(\Delta t_n)^{-1} = T_n^{-1} = \frac{\Delta E_n}{h}. \tag{1}$$

Evidently in virtue of (32) the emission time rate of the energy ΔE_n is equal to

$$\frac{\Delta E_n}{\Delta t_n} = \frac{(\Delta E_n)^2}{h}. \tag{2}$$

This rate can be compared with quantum-mechanical results [15]. Since (2) is proportional to $(\Delta E_n)^2$ in table 1 we present the factors entering $(\Delta E)^2$ dependent on n, i.e. the expressions

$$(\Delta E_n)^2 \sim \left| \frac{1}{(n+1)^2} - \frac{1}{n^2} \right|^2 \tag{3}$$

calculated according to (17), for the levels beginning with n = 1 up to n = 5.

The quantum-mechanical results for the total transition probabilities between the quantum states

$$(n+1)p - ns, \tag{4}$$

Where n = 1,2,3,4 and 5 and probabilities are expressed in 10^8 sec^{-1} units, are [15]:

$$22p - 1s \quad 6.25 \tag{5}$$

$$3p - 2s \quad 0.22 \tag{6}$$

$$4p - 3s \quad 0.030 \tag{7}$$

$$5p - 4s \quad 0.0075 \tag{8}$$

$$6p - 5s \quad 0.0021 \tag{9}$$

In table 2 the ratios of (3), calculated for different pairs of n and n+1, are compared with the ratios of the quantum-mechanical data obtained for the same pairs of n and n+1 entering (5)-(9). A parallelism of the quantum-mechanical results with those calculated from the present semiclassical theory is evident.

Table 1: Expression (3) entering $(\Delta E_n)^2$ in (2) calculated for different quantum levels n.

n	Expression (B3)
1	$(3/4)^2$
2	$(5/16)^2$
3	$(7/144)^2$
4	$(9/400)^2$
5	$(11/900)^2$

Table 2: Ratios of expressions (B3) entering table 1 calculated for different pairs of the level indices n and n+1 compared with the ratios of the transition intensity data given by the formulae (B5)-(B9).

n	n+1	Indices ratio	Ratios of expressions (B3)	Ratios of probabilities from the eqs. (B5)-(B9)
1	2	1:2	$(3/4)^2 : (5/36)^2 = 28.7$	$(B5) : (B6) = 6.25:0.22=28.4$
2	3	2:3	$(5/36)^2 : (7/144)^2 = 8.16$	$(B6) : (B7) = 0.22:0.30=7.33$
3	4	3:4	$(7/144)^2 : (9/400)^2 = 4.74$	$(B7) : (B8) = 0.30:0.0075=4.00$
4	5	4:5	$(9/400)^2 : (11/900)^2 = 3.40$	$(B8) : (B9) = 0.0075:0.0021=3.5$