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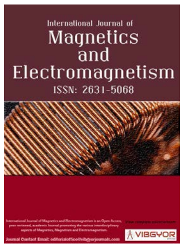

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Is a Propagating Infinite Plane Wave a "Radiation Field?"

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Abstract

The total electromagnetic field of the Hertzian dipole has components that decrease with distance, *r*, as $1/r^3$, $1/r^2$, and $1/r$. The latter component is called the "radiation field" and represents energy lost from the source. For finite-size current sources, the total distant fields can be calculated by summing just the radiation field component of the Hertzian dipoles comprising the source. We show mathematically that the propagating plane wave radiated by an infinite planar current source, composed of Hertzian dipoles, involves all field components. The nature of this paper is pedagogical: It is to clarify for electromagnetics students the semantic issues arising from discussions on radiation from antennas, in particular, radiating fields vs. radiation fields.

Introduction

The electric and magnetic fields of the Hertzian dipole, an infinitesimal time-varying current-carrying element with opposite charges residing at either end (via the continuity equation), are fundamental to the understanding of the fields of time-varying current and charge sources and, in particular, in solving for field patterns from current sources such as the half-wavelength antenna and various antenna arrays. That is, given the fields of the Hertzian dipole, one can sum (integrate) the fields of groupings of these infinitesimal current elements to determine the fields of any current source.

The standard approach to solving for the fields of the Hertzian dipole, found in most textbooks [1,2], is to recast Maxwell's Equations in the frequency domain in terms of a vector potential $\vec{A}(\vec{r})$ and a scalar potential $\Phi(\vec{r})$, as

$$\nabla^2\vec{A} + \omega^2\mu\in\vec{A} = -\mu\vec{J} \tag{1}$$

$$\nabla^2\Phi + \omega^2\mu\in\Phi = -\frac{\rho}{\in} \tag{2}$$

$$\nabla\cdot\vec{A} + j\omega\Phi = 0 \tag{3}$$

Where

$$\vec{E} = -\nabla\Phi - j\omega\vec{A} \tag{4}$$

$$\vec{B} = \nabla\times\vec{A} \tag{5}$$

Equations (1) and (2) are inhomogeneous wave equations, and eq. (3) is the Lorenz condition. $\vec{B}(\vec{r})$ is the magnetic flux density, $\vec{E}(\vec{r})$ the electric field intensity, $\vec{J}(\vec{r})$ the volume current density $\rho(\vec{r})$ the volume charge density, ω the angular frequency, μ the medium permeabililty and $\in$ the

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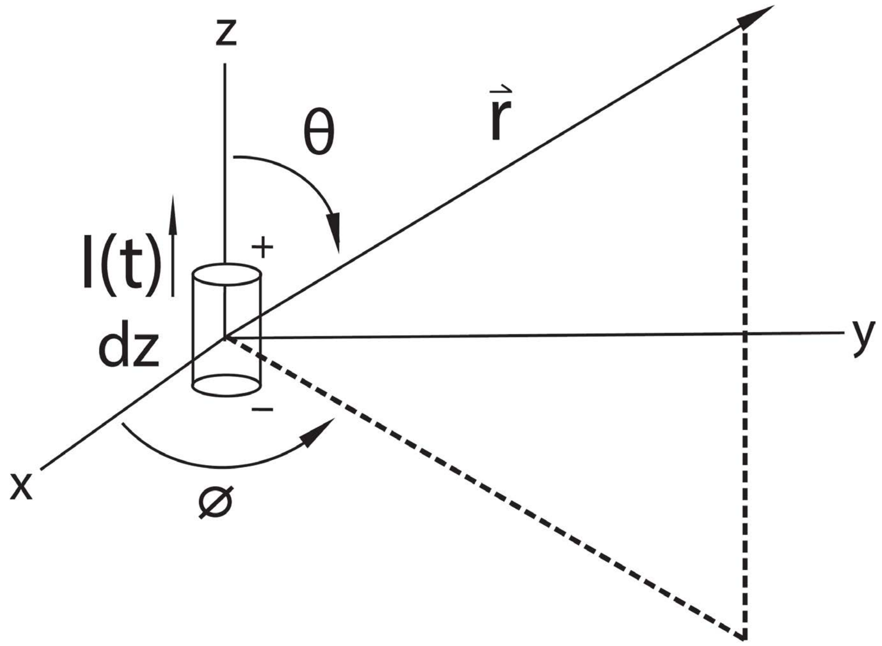

Figure 1: Hertzian dipole at the origin.

medium permittivity. "j" is equal to the square root of (-1).

For the case of an infinitesimal current source, the solution to $d\vec{A}$ in eqs. (1-5) is

$$d\vec{A} = \frac{\mu I e^{j\omega\left(t - \sqrt{\mu\epsilon} r\right)}}{4\pi} \hat{z} dz \tag{6}$$

for the source at the origin of a spherical coordinate system and oriented along the z-axis with length of *dz*, as shown in Figure 1. Application of eqs. (3-5) to (6) yields [1,2].

$$B_\phi = \frac{\mu I dz}{4\pi} \sin\theta \left[\frac{jk}{r} + \frac{1}{r^2}\right] e^{j(\omega t - kr)} \tag{7}$$

$$B_r = B_\theta = 0 \tag{8}$$

$$E_r = \frac{\eta I dz}{2\pi} \cos\theta \left[\frac{1}{r^2} - \frac{j}{kr^3}\right] e^{j(\omega t - kr)} \tag{9}$$

$$E = \frac{Idz}{} \sin \left[\frac{jk}{r} + \frac{}{r^2} - \frac{j}{kr^3}\right] e^{j(\ t - kr)} \tag{10}$$

$$E_\phi = 0 \tag{11}$$

Hence, the fields at *r* are delayed a time r/c from the time of the current, with $k = \omega/c$, and wave speed, *c*, equal to $1/\sqrt{\mu\in}$. The spherical coordinates *r*, *ϑ*, and *φ* represent, respectively, a radial distance from the origin and the zenith and azimuth angles between the origin and *r*. It is traditional to identify each of the terms in these equations by their radial variation from the source. The $1/r^3$ term is referred to as the "electrostatic field" and is present only in the electric field. It dominates the field at close distances, specifically where *r* is much less than the wavelength $\lambda = 2\pi c/\omega$, and represents the field of a time-varying point dipole. The $1/r^2$ dependent field, present in B_φ, B_ϑ, and E_r, is called the "induction field" and it, along with the electrostatic terms are called the "near fields." In the case of the magnetic induction, the $1/r^2$ term represents the time-varying Biot-Savart law. The final terms, dependent on $1/r$, are called the "radiation fields." The radiation fields are responsible for carrying energy, or radiating, away from the source and are the dominant fields where *r* is much greater than the wavelength. For calculating the fields of antennas on the order of wavelength size, the spatial integration of the fields of the infinitesimal dipoles, as noted earlier, need only involve the radiation fields of the source dipoles. Antenna-array field-pattern computations similarly need only consider the radiation fields. If one is far enough away from a source, or, more specifically, if the radius of curvature of the wave-front is large compared to the receiver dimension, it is a reasonable approximation to consider the spherical wave-front generated by the source to be essentially planar, although it decreases as $1/r$ and is not the pure or exact uniform plane wave derivable from Maxwell's Equations in Cartesian coordinates, discussed next.

The "source-free" uniform plane wave whose fields are given in eqs. (12-14) is commonly derived in Cartesian coordinates from the homogeneous Helmholtz wave equation, which is itself derived from the Maxwell curl equations with $\rho(\vec{r}) = \vec{\boldsymbol{J}}(\vec{r}) = 0$, giving

$$E_z(x) = -E_0 e^{j(\omega t - kx)} \quad (12)$$

$$H_y(x) = -H_0 e^{j(\omega t - kx)} \quad (13)$$

$$\frac{E_z}{H_y} = \sqrt{\frac{\mu}{\in}} = \eta \quad (14)$$

Which represent a sinusoidally oscillating transverse wave with orthogonal electric, $E_z(x)$, and magnetic, $H_y(x)$, components traveling at the speed of light, c, in the x-direction [1,2]. No comment is made in common textbooks regarding the means by which this plane wave could be produced without a source current or, if a source current is needed, what that source current might be. Using the Hertzian dipole approach discussed above, there must be a set of infinitesimal current sources whose arrangement produces the plane wave pattern of eqs. (12-14). That this is the case may seem intuitively unreasonable, since the infinitesimal dipole source has component fields that fall off as $1/r^3$, $1/r^2$, and $1/r$, the latter being called the "radiation field," whereas the "source-free" radiating plane wave of eqs. (12-14) is sinusoidally constant in amplitude with distance. Hence, the question found in this paper's title. In the next section, it is shown that multiple Hertzian dipoles can indeed be arranged to be the source of a uniform plane wave, but all dipole field components must be included in the proper calculation of the fields of the plane wave.

The Infinite Array of Hertzian Dipoles

To answer the question found in the title of this paper, we begin with the Cartesian expression for $\vec{\boldsymbol{B}}(\vec{\boldsymbol{r}})$ of eq. (7), written in terms of its induction and radiation components

$$d\vec{\boldsymbol{B}}_{ind} = \frac{\mu_0 K_z dy'dz' e^{-jk\sqrt{(x)^2 + (y-y')^2 + (z-z')^2}}}{4\pi\sqrt{(x)^2 + (y-y')^2 + (z-z')^2}^3}\left[-\hat{x}(y-y') + \hat{y}x\right] \quad (15)$$

$$d\vec{\boldsymbol{B}}_{rad} = \frac{jk\mu_0 K_z dy'dz' e^{-jk\sqrt{(x')^2 + (y-y')^2 + (z-z')^2}}}{4\pi\left((x)^2 + (y-y')^2 + (z-z')^2\right)}\left[-\hat{x}(y-y') + \hat{y}x\right] \quad (16)$$

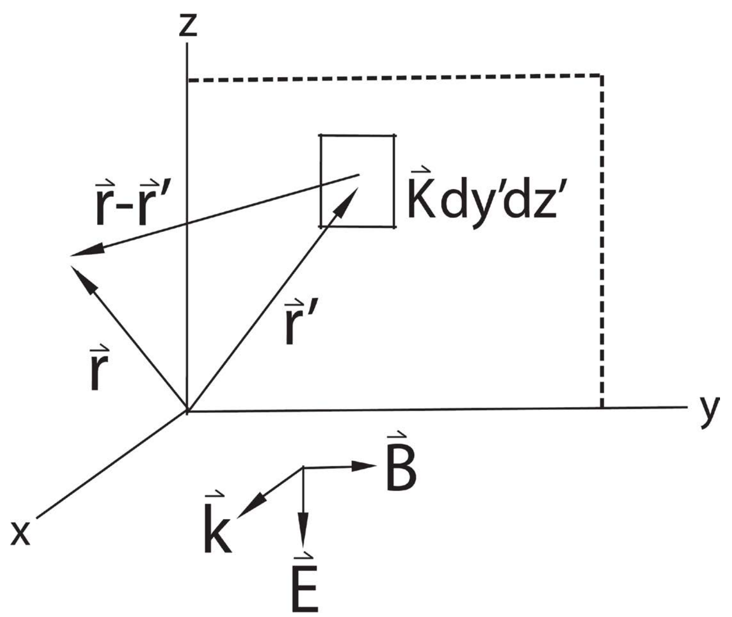

Figure 2: Infinite current sheet.

as derived in Appendix A. The $K_z dy'dz'$ factor is the infinitesimal surface current equivalent of *Idz*, and it will be integrated over a large flat conductor in the yz-plane, located at $x' = 0$, as shown in Figure 2. As follows from eq. (15-16) and Figure 3, any $\vec{B}$-field component in the $\pm x$ direction is canceled by a $\mp x$ directed component from a little piece of current equidistant in the opposite direction from the field point. It follows that the field components may then be written as

$$\vec{B}_{ind} = \hat{y}\frac{\mu_0 K_z x}{4\pi}\int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dx' \frac{e^{-jk\sqrt{x^2+(y-y')^2+(z-z')^2}}}{\sqrt{x^2+(y-y')^2+(z-z')^2}^{\,3}} \quad (17)$$

$$\vec{B}_{rad} = \hat{y}\frac{jk\mu_0 K_z x}{4\pi}\int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dx' \frac{e^{-jk\sqrt{x^2+(y-y')^2+(z-z')^2}}}{x^2+(y-y')^2+(z-z')^2} \quad (18)$$

Where the origin may be placed anywhere on the infinite sheet, so we let *y* = *z* = 0. A cylindrical substitution, $y'^2 + z'^2 = \rho'^2$, $dy'dz' = \rho' d\rho' d\phi'$ is then made to facilitate the integration. The new limits on the integral(s) are $0 < \rho' < \infty$ and $0 < \phi < 2\pi$. The factor $\hat{y}\frac{\mu_0 K_z x}{4\pi}$ in both terms will be defined as $\hat{n}$. Thus

$$\vec{B}_{ind} = \hat{n}\int_0^{2\pi} d\phi' \int_0^{\infty} \rho' d\rho' \frac{e^{-jk\sqrt{\rho'^2+x^2}}}{\sqrt{\rho'^2+x^2}^{\,3}} \quad (19)$$

$$\vec{B}_{rad} = jk\hat{n}\int_0^{2\pi} d\phi' \int_0^{\infty} \rho' d\rho' \frac{e^{-jk\sqrt{\rho'^2+x^2}}}{\rho'^2+x^2} \quad (20)$$

(21)

If we let $u = \rho'^2 + x^2$, and replace the limits, the integrals become

$$\vec{B}_{ind} = 2\pi\hat{n}\int_x^{\infty} du \frac{e^{-jku}}{u^2} \quad (22)$$

$$\vec{B}_{rad} = 2\pi jk\hat{n}\int_x^{\infty} du \frac{e^{-jku}}{u} \quad (23)$$

(24)

Which can be found in integral tables [3]. The above integrals yield

$$\vec{B}_{ind} = \lim_{u\to\infty} 2\pi\hat{n}\left(\frac{-e^{-jku}}{u} - jkEi(-jku) - \left(\frac{-e^{-jkx}}{x} - jkEi(-jkx)\right)\right) \quad (25)$$

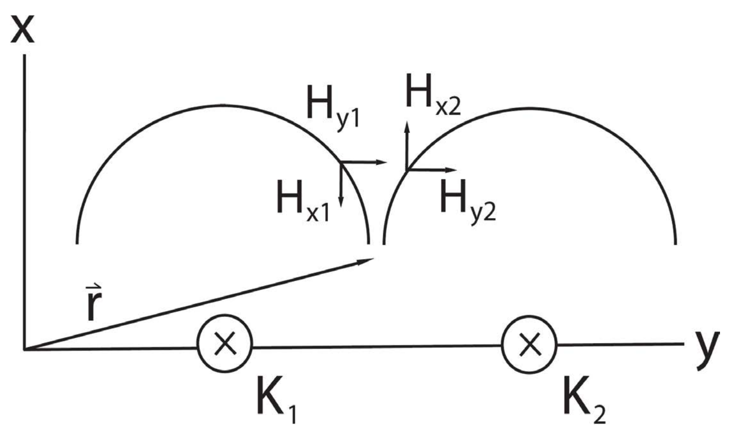

Figure 3: The B_x component from any piece of current cancels with that of an adjacent piece.

$$\vec{\boldsymbol{B}}_{rad} = \lim_{u\to\infty} 2\pi\hat{n}jk\left(Ei\left(-jku\right) - Ei\left(-jkx\right)\right) \tag{26}$$

Where,

$$Ei\left(av\right) = \int \frac{e^{av}}{v}dv \tag{27}$$

is the exponential integral [3]. In summing eq. (25) and eq. (26), the two exponential integral terms from each equation cancel one another, and as $u \to \infty$, the first term in eq. (25) goes to zero, leaving only the third term in $\vec{\boldsymbol{B}}_{ind}\left(\vec{\boldsymbol{r}}\right)$. When we replace $\hat{n}$ by its definition

$$\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_{ind} + \vec{\boldsymbol{B}}_{rad} = \hat{y}\frac{\mu_0 K_z}{2}e^{-jkx} \tag{28}$$

and using Maxwell's equation $\nabla \times \vec{\boldsymbol{H}} = j\omega \in \vec{\boldsymbol{E}}$ gives

$$\vec{\boldsymbol{E}}\left(\vec{\boldsymbol{r}}\right) = -\hat{z}\frac{\mu_0 K_z c}{2}e^{-jkx} \tag{29}$$

Which is equivalent to eqs. (12-13) with

$$H_0 = \frac{K_z}{2} \tag{30}$$

$$E_0 = \frac{\mu_0 K_z c}{2} = \sqrt{\frac{\mu_0}{\epsilon_0}}\frac{K_z}{2} \tag{31}$$

being the amplitudes of the wave components in eqs. (12-13) and η keeping its definition from eq. (14). It is clear from the summation of eqs. (25) and (26), that both the radiation and the induction fields of the dipoles are involved in producing the uniform plane wave (whose amplitude does not decay with distance). Thus, the radiating plane wave is not a radiation field.

Discussion and Conclusion

According to the calculations of section II, the radiation field components of the dipoles comprising the plane wave source do not solely make up the fields that propagate; Parts of the magnetic induction field cancel parts of the magnetic radiation field. Presumably, the electric field of the radiating plane wave is composed of all three dipole fields components.

The fact that all dipole field components are involved in the plane wave is a consequence of the infinite size of the source. To be in the far-field of all of the dipoles together would require being an infinite distance from the source.

Although a current source of infinite extent is not practical, some reasonable approximate plane-wave-generating devices have been constructed from multiple dipoles [4]. The primary technical requirement is in the phasing of each dipole which makes up the sheet. Each dipole needs to have an identical current, in both magnitude and phase.

An alternate and simpler, but less pedagogically satisfying derivation of the plane wave from the infinite current sheet, using $d\vec{A}$ directly, is given in Appendix B.

Finally, the calculation of the fields of an infinite current source, as given in the body of this paper and in Appendix B, is not commonly found in textbooks and probably should be, given that it illuminates the nature of the seemingly understood definitions of radiation and radiating.

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Appendix

Appendix A: From spherical to cartesian

To convert eq. (7) from spherical to Cartesian, we begin with a redefining of

$$r \rightarrow R = |\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'| \quad \text{(A1)}$$

Where

$$\vec{\boldsymbol{r}} = x\hat{x} + y\hat{y} + z\hat{z} \quad \text{(A2)}$$

$$\vec{\boldsymbol{r}}' = x'\hat{x} + y'\hat{y} + z'\hat{z} \quad \text{(A3)}$$

Which is depicted in Figure 2 for the infinite current sheet. Using eq. (A1) may rewrite eq. (7) as

$$\vec{\boldsymbol{B}} = \frac{\mu I dl'}{4\pi} sin\theta \left[\frac{jk}{R} + \frac{1}{R^2} \right] e^{-jkR} \hat{\phi} \quad \text{(A4)}$$

Which is not longer necessarily centered at the spatial origin. The definition of φ and ϑ then must be chosen so that they are the azimuth and zenith angles which lie between source and observation coordinates during the integration. Those definitions give

$$\sin \quad = \frac{\sqrt{(x - x')^2 \quad (y - y')^2}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} = — \quad \text{(A5)}$$

$$\hat{\phi} = - \sin(\phi)\hat{x} + \cos(\phi)\hat{y} \quad \text{(A6)}$$

Where

$$\sin(\phi) = \frac{y - y'}{\sqrt{(x - x')^2 + (y - y')^2}} = \frac{y - y'}{\rho} \quad \text{(A7)}$$

$$\cos(\phi) = \frac{x - x'}{\sqrt{(x - x')^2 + (y - y')^2}} = \frac{x - x'}{\rho} \quad \text{(A8)}$$

Substituting the above into eq. (A4) yields

$$\vec{\boldsymbol{B}}(x,y,z) = \frac{\mu I dl'}{4\pi} \frac{\rho}{R} \left[\frac{jk}{R} + \frac{1}{R^2} \right] e^{-jkR} \left(- \frac{y - y'}{\rho}\hat{x} + \frac{x - x'}{\rho}\hat{y} \right) \quad \text{(A9)}$$

Canceling terms and writing $\vec{\boldsymbol{B}}$ as a differential in its induction and radiation components we find

$$d\vec{\boldsymbol{B}}_{ind} = \frac{\mu I dl'}{4\pi} \left[\frac{1}{R^3} \right] e^{-jkR} \left[-\hat{x}(y - y') + \hat{y}(x - x') \right] \quad \text{(A10)}$$

$$d\vec{\boldsymbol{B}}_{rad} = \frac{\mu I dl'}{4\pi} \left[\frac{jk}{R^2} \right] e^{-jkR} \left[-\hat{x}(y - y') + \hat{y}(x - x') \right] \quad \text{(A11)}$$

Recalling the definition of *R* from above, we convert eq. (A10-A11) to

$$d\vec{\boldsymbol{B}}_{ind} = \frac{\mu I dl'}{4\pi} \left[\frac{e^{-jk\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}^{\,3}} \right] \left[-\hat{x}(y - y') + \hat{y}(x - x') \right] \quad \text{(A12)}$$

$$d\vec{\boldsymbol{B}}_{rad} = \frac{\mu I dl'}{4\pi} \left[\frac{jk\, e^{-jk\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}}{(x - x')^2 + (y - y')^2 + (z - z')^2} \right] \left[-\hat{x}(y - y') + \hat{y}(x - x') \right] \quad \text{(A13)}$$

Appendix B: The infinite current sheet, magnetic vector potential, and plane wave

An infinite sheet of current is assumed, identical to that of section II, and $\vec{A}$ is from eq. (6). The vector potential, A_z, which by eq. (6) is the only component of $\vec{A}$ present, will be calculated in cylindrical coordinates, where ρ is a radial distance fixed to the y-z plane, φ is the azimuth angle, and x is the vertical dimension. The expression for $\vec{A}$ is then

$$\vec{A} = -\hat{z}\frac{\mu_0 K_z}{4\pi}\int_0^L \rho' d\rho' \int_0^{2\pi} d\phi' \frac{e^{-jk\sqrt{\rho'^2+x^2}}}{\sqrt{\rho'^2+x^2}} \quad \text{(B1)}$$

in which L will be taken to infinity to cover the whole y-z plane. From the φ integration

$$A_z = -\frac{\mu_0 K_z}{2}\int_0^L \rho' d\rho' \frac{e^{-jk\sqrt{\rho'^2+x^2}}}{\sqrt{\rho'^2+x^2}} \quad \text{(B2)}$$

Which leads to

$$A_z = -\frac{\mu_0 K_z j}{2k}\left[e^{-jk\sqrt{L^2+x^2}} - e^{-jkx}\right] \quad \text{(B3)}$$

By eq. (5)

$$\vec{B} = -\hat{y}\frac{\partial A_z}{\partial x} \quad \text{(B4)}$$

$$-\hat{y}\frac{\partial A_z}{\partial x} = \hat{y}\frac{\mu_0 K_z}{2}\left[-x\frac{e^{-jk\sqrt{L^2+x^2}}}{\sqrt{L^2+x^2}} + e^{-jkx}\right] \quad \text{(B5)}$$

As L is taken to infinity the first term on the right hand side goes to zero, leaving

$$\vec{B} = \hat{y}\frac{\mu_0 K_z}{2}e^{-jkx} \quad \text{(B6)}$$

In source-free space, Ampere's law may be written in the frequency domain as

$$\vec{E} = \frac{c^2}{j\omega}\nabla \times \vec{B} \quad \text{(B7)}$$

Which results in only a component of $\vec{E}$ in the $-\hat{z}$-direction; namely

$$\vec{E} = -\hat{z}\frac{\mu_0 K_z c}{2}e^{-jkx} \quad \text{(B8)}$$

This approach to finding the plane wave from an infinite sheet of current is clearly more straightforward than the procedures of section II, but not nearly as pedagogically satisfying in terms of clarifying the relative importance of individual field components in the generation of a uniform plane wave.