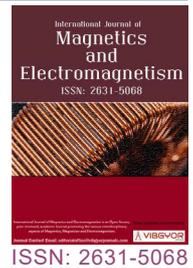


Extending Weber's Electrodynamics to High Velocity Particles



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Abstract

Weber's electrodynamics has been shown to explain electromagnetic phenomena by a number of researchers. It has a much simpler form for particle-particle interaction than the popularly used Maxwell equations. However, it has also been shown that Weber's electrodynamics got its limitations. It is not suitable for high velocity particles. And it had negative mass issue, causing non-physical explanation in some circumstances. In this article, we introduce a new theory of weber-like electrodynamics, trying to address the shortcomings of Weber's electrodynamics, and extending Weber's electrodynamics to high velocity particles.

Introduction

Weber's electrodynamics has been shown to explain electromagnetic phenomena, such as longitudinal force inside electric wire [1,2], anomalous diffusion in plasmas [3], deriving gravitation from electromagnetism [4], electric and magnetic fields surrounding current [5,6], the London moment and Meissner effect of superconductivity [7], and electromagnetic induction [8,9], etc. Weber's electrodynamics has a much simpler form for particle-particle interaction than the popularly used Maxwell equations and Lorentz force [10]. Moreover, Weber's electrodynamics explains the outcome of two recent electron beam experiments better than Maxwell-Lorentz electrodynamics [11,12]. For applications involving large quantity of particles, a new field theory based on Weber's electrodynamics has been introduced [13].

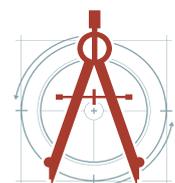
However, Weber's electrodynamics had been criticized for the negative mass issue by Helmholtz shortly after its introduction [14]. Inside a non-conducting charged spherical shell, a moving particle may gain a negative mass and its velocity may go without limit according to Weber's electrodynamics. Phipps tried to address this issue by introducing a modified form of Weber's electrodynamics [15]. Nevertheless, it was shown that Weber's electrodynamics does not apply to high velocity particles [16]. Particles may be accelerated to a velocity higher than light speed, contradicting the outcome of linear accelerator experiments [17]. Phipps's modified form of Weber's electrodynamics also suffers from this problem [18]. Later, the issue of unlimited velocity seems to have been solved by utilizing Schrodinger's kinetic energy instead of Newtonian kinetic energy [14,19]. Weber's electrodynamics plus Schrodinger's kinetic energy predict a particle oscillation pattern beside a capacitor that is different from the relativistic mechanics [19].

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In this article, we introduce a new Weber-like theory of velocity-dependent potential energy and kinetic energy, which can be applied to particles of velocity close to light speed. Using this velocity-dependent potential energy, we can get a weber-like electrodynamic force. This new Weber-like theory may be seen as an extension of Weber's electrodynamics.

Weber-Like Potential Energy

Let's consider a two-body system along x-axis in a laboratory rest frame (Figure 1). Two charged particles (one with mass m_1 , charge Q , and the other with mass m_2 , charge q) are at rest and in a distance r_0 from each other (Figure 1a). The potential energy of the two-body system is:

$$U_0 = \frac{Qq}{4\pi\epsilon_0 r_0} \quad (1)$$

Where ϵ_0 is the permittivity. Because of the electric expelling force, the particles move away from each other. At one time point, they are in a distance r from each other, and their velocities become v_1 and v_2 respectively (Figure 1b). We postulate that the potential energy of the two-body system becomes:

$$U_r = \frac{Qq}{4\pi\epsilon_0 r} \sqrt{1 - \frac{(v_1 - v_2)^2 c^2}{(c^2 - v_1 \cdot v_2)^2}} \quad (2)$$

Where c is light speed. In this expression, the potential energy is velocity dependent. If $v_1 = v_2$, the potential energy is simplified to Coulomb potential:

$$U_r = \frac{Qq}{4\pi\epsilon_0 r} \quad (3)$$

If $-v_1 = v_2 \rightarrow c$, then $|v_1 - v_2| \rightarrow 2c$. The equation (2) still holds, and can be simplified to:

$$U_r = \frac{Qq}{4\pi\epsilon_0 r} \sqrt{1 - \frac{(v_1 - v_2)^2}{4c^2}} \quad (4)$$

If $|v_1 \cdot v_2|$ is much less than c^2 , the equation (2) can be simplified to:

$$U_r = \frac{Qq}{4\pi\epsilon_0 r} \sqrt{1 - \frac{(v_1 - v_2)^2}{c^2}} \quad (5)$$

Except being one-dimensional, this expression is the same as the potential proposed by Phipps [15].

If $(v_1 - v_2)^2$ is much less than c^2 , we can simplify equation (5) further by only keeping the first and second order terms:

$$U_r = \frac{Qq}{4\pi\epsilon_0 r} \left(1 - \frac{(v_1 - v_2)^2}{2c^2} \right) \quad (6)$$

Except being one-dimensional, this expression is exactly the same as the Weber's potential [10]. Thus the potential energy in this article can be seen as an extension of Weber's potential.

New Kinetic Energy and Momentum

We postulate new kinetic energy and momentum expressions by assuming a velocity dependent force-acceleration relationship:

$$F = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \ddot{a} \quad (7)$$

Where m , v , F , and \ddot{a} are a particle's mass, velocity, external force and acceleration respectively. The

velocity dependency in equation (7) looks similar to relativistic mass [20]. However, here it is a coefficient of acceleration for force, instead of coefficient of velocity for momentum as in Special Relativity theory [21]. The new expressions of kinetic energy and moment can be derived:

$$K = \int_0^x F dx = \int_0^x \frac{m}{1 - \frac{v^2}{c^2}} \ddot{a} dx = \int_0^v \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} dv = mc^2 \left(1 - \sqrt{1 - \frac{v^2}{c^2}} \right)$$

$$p = \int_0^t F dt = \int_0^t \frac{m}{1 - \frac{v^2}{c^2}} \ddot{a} dt = \int_0^v \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} dv = mc \sin^{-1} \frac{v}{c} \quad (8)$$

When both particles are at rest in a laboratory rest frame (Figure 1a), the kinetic energy and momentum of the two-body system equal zero. When the two particles' velocities become v_1 and v_2 respectively (Figure 1b), we postulate that the kinetic energy and momentum of the two-body system become:

$$K = (m_1 + m'_1)c^2 \left(1 - \sqrt{1 - \frac{v_1^2}{c^2}} \right) + (m_2 + m'_2)c^2 \left(1 - \sqrt{1 - \frac{v_2^2}{c^2}} \right)$$

$$p = (m_1 + m'_1)c \sin^{-1} \frac{v_1}{c} + (m_2 + m'_2)c \sin^{-1} \frac{v_2}{c} \quad (9)$$

Where m'_1 and m'_2 are the mass additional to the original mass m_1 and m_2 . The original mass is the particle's mass when the particle is at rest and has no interacting nearby particles.

When two particles are far away from each other, the potential energy is fully converted to kinetic energy. The two particles achieved velocity V_1 and V_2 respectively. To calculate m'_1 and m'_2 in equation (9), we postulate these equations below:

$$m_1 + m'_1 = \frac{m_1}{\sqrt{1 - \frac{v_1^2}{c^2}}}$$

$$m_2 + m'_2 = \frac{m_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} \quad (10)$$

To calculate V_1 and V_2 , we use equations of conservation of energy and conservation of momentum. The kinetic energy of two particles equal the initial potential energy, and the momentum of two particles equal zero.

$$\frac{m_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} c^2 \left(1 - \sqrt{1 - \frac{v_1^2}{c^2}} \right) + \frac{m_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} c^2 \left(1 - \sqrt{1 - \frac{v_2^2}{c^2}} \right) = U_0$$

$$\frac{m_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} c \sin^{-1} \frac{v_1}{c} + \frac{m_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} c \sin^{-1} \frac{v_2}{c} = 0 \quad (11)$$

From equations (10,11), we can see that additional mass are independent to particle's velocity v_1 and v_2 . Instead, they are determined with particles final velocity V_1 and V_2 when potential energy is fully converted to kinetic energy. After rearranging equations (10,11), we can get:

$$m'_1 + m'_2 = \frac{U_0}{c^2} \tag{12}$$

Thus the additional mass is also related to the initial potential energy U_0 , and can be seen as a constant, which does not change with particle's velocity. The initial potential energy and the sum of additional mass satisfy the energy-mass relationship, which was first introduced by Special Relativity theory [21].

Velocity and Energy of an Accelerated Particle

Let's consider the two-body system along x-axis in a laboratory rest frame (Figure 1). The two particles are at rest at the beginning, then their velocities become v_1 and v_2 respectively. According to energy conservation, the decrease of potential energy equals the increase of kinetic energy. From the equations (1), (2), and (9), we get:

$$\begin{aligned} \frac{Qq}{4\pi\epsilon_0 r_0} - \frac{Qq}{4\pi\epsilon_0 r} \sqrt{1 - \frac{(v_1 - v_2)^2 c^2}{(c^2 - v_1 \cdot v_2)^2}} \\ = (m_1 + m'_1)c^2 \left(1 - \sqrt{1 - \frac{v_1^2}{c^2}} \right) + (m_2 + m'_2)c^2 \left(1 - \sqrt{1 - \frac{v_2^2}{c^2}} \right) \end{aligned} \tag{13}$$

The momentum of the two-body system (Equation 9) always equals zero, according to momentum conservation. Thus, if we assume that m_1 is way larger than m_2 , particle Q will stay at rest, i.e. v_2 and V_1 approximately equal zero. Using Equations (10, 12), we can get that $m'_1 = 0$ and $m'_2 = \frac{U_0}{c^2}$. Thus, Equation (13) can be simplified to:

$$\frac{Qq}{4\pi\epsilon_0 r_0} - \frac{Qq}{4\pi\epsilon_0 r} \sqrt{1 - \frac{v_2^2}{c^2}} = \left(m_2 + \frac{Qq}{4\pi\epsilon_0 r_0 c^2} \right) c^2 \left(1 - \sqrt{1 - \frac{v_2^2}{c^2}} \right) \tag{14}$$

The above equation shows that the particle velocity v_2 can be predicted given the initial Coulomb potential $\frac{Qq}{4\pi\epsilon_0 r_0}$ and the current Coulomb potential $\frac{Qq}{4\pi\epsilon_0 r}$. The particle velocity v_2 can approach, but not exceed light speed (Figure 2).

This two-body system may be seen as a greatly simplified representation of the linear accelerator experiment [17]. The stationary particle Q represents the stationary accelerator instrument that creates

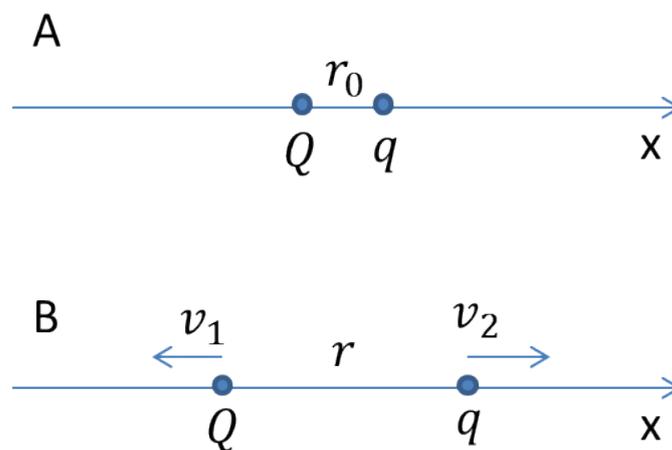
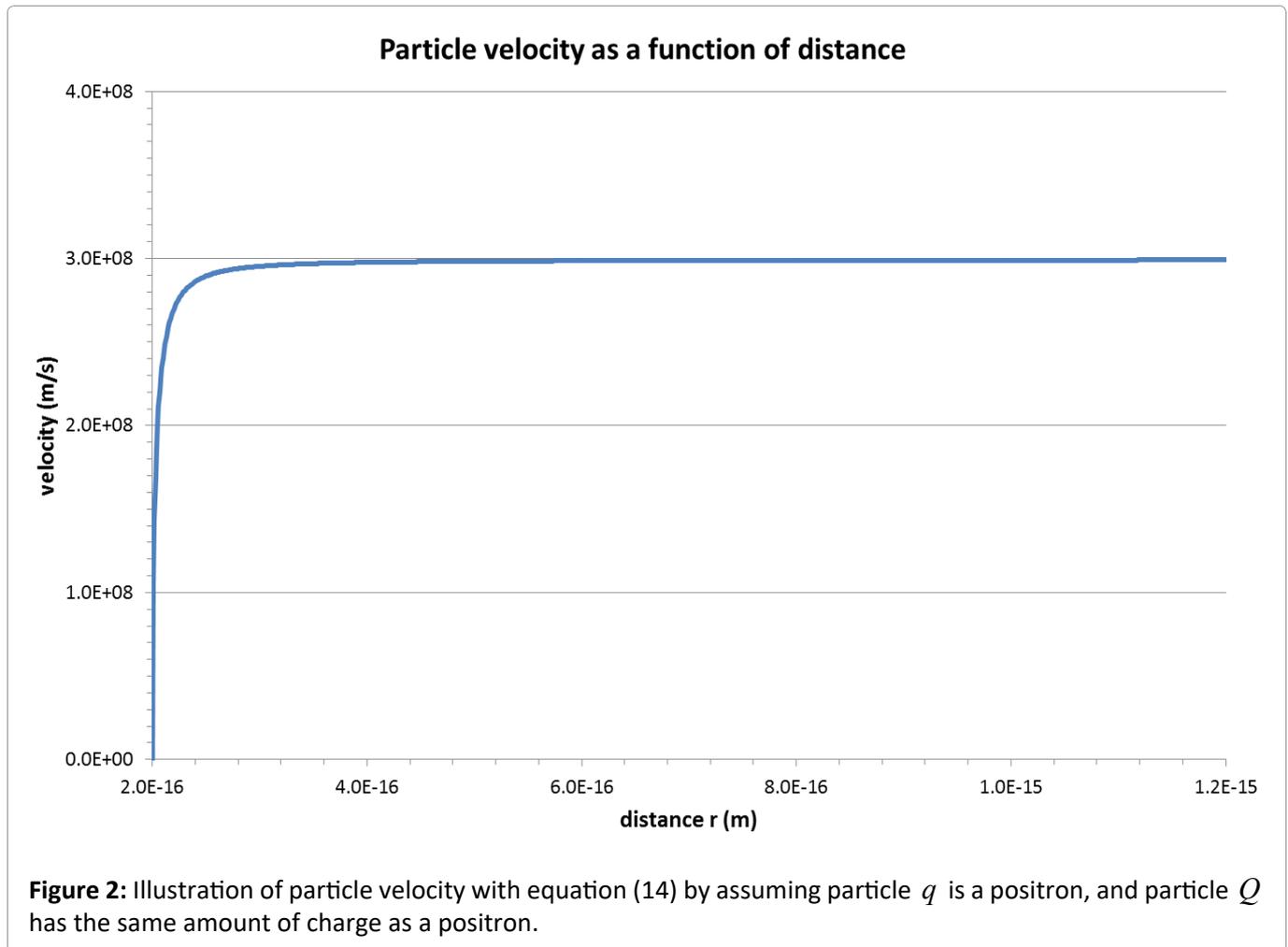


Figure 1: A) Both charge Q and charge q are at rest and in a distance of r_0 from each other; B) Two charges are in a distance of r from each other, with velocity v_1 and v_2 respectively.



electric potential, while the moving particle q represents an accelerated electron.

Equation (14) can be re-arranged as:

$$\frac{Qq}{4\pi\epsilon_0 r_0} - \frac{Qq}{4\pi\epsilon_0 r} = m_2 c^2 \left(\frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}} - 1 \right) \tag{15}$$

The above equation is the same as that would be given by Special Relativity theory [21]. The left side of the equation is the Coulomb potential energy change and the right side is the relativistic kinetic energy. Thus the velocity prediction of the new Weber-like theory would match to the linear accelerator experiment [17] the same as Special Relativity theory does.

Even though the predicted particle velocity is the same as that of Special Relativity theory, the potential and kinetic energy of the two theories are very different. To fully illustrate this difference, let's consider a charge q moving along x-axis, which is accelerated by one stationary charge Q_1 . It is then approaching another stationary charge Q_2 and being de-accelerated (Figure 3).

In the Special Relativity theory, the particle gains kinetic energy during acceleration. And kinetic energy is converted back to potential energy during de-acceleration. The Special Relativity theory states that the conversion amount is independent to particle velocity, instead only dependent on initial position r_0 and current position r (Figure 4).

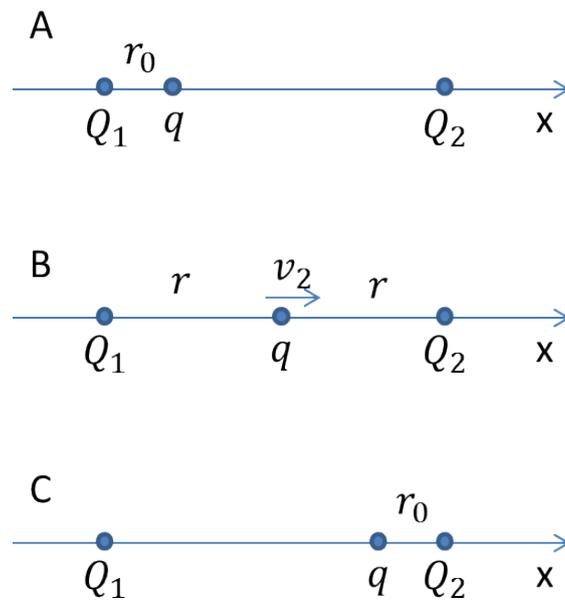


Figure 3: Both charges Q_1 and Q_2 are always at rest: A) The charge q is at rest; B) The charge q is accelerated to velocity v_2 ; C) The charge q is deaccelerated to rest again.

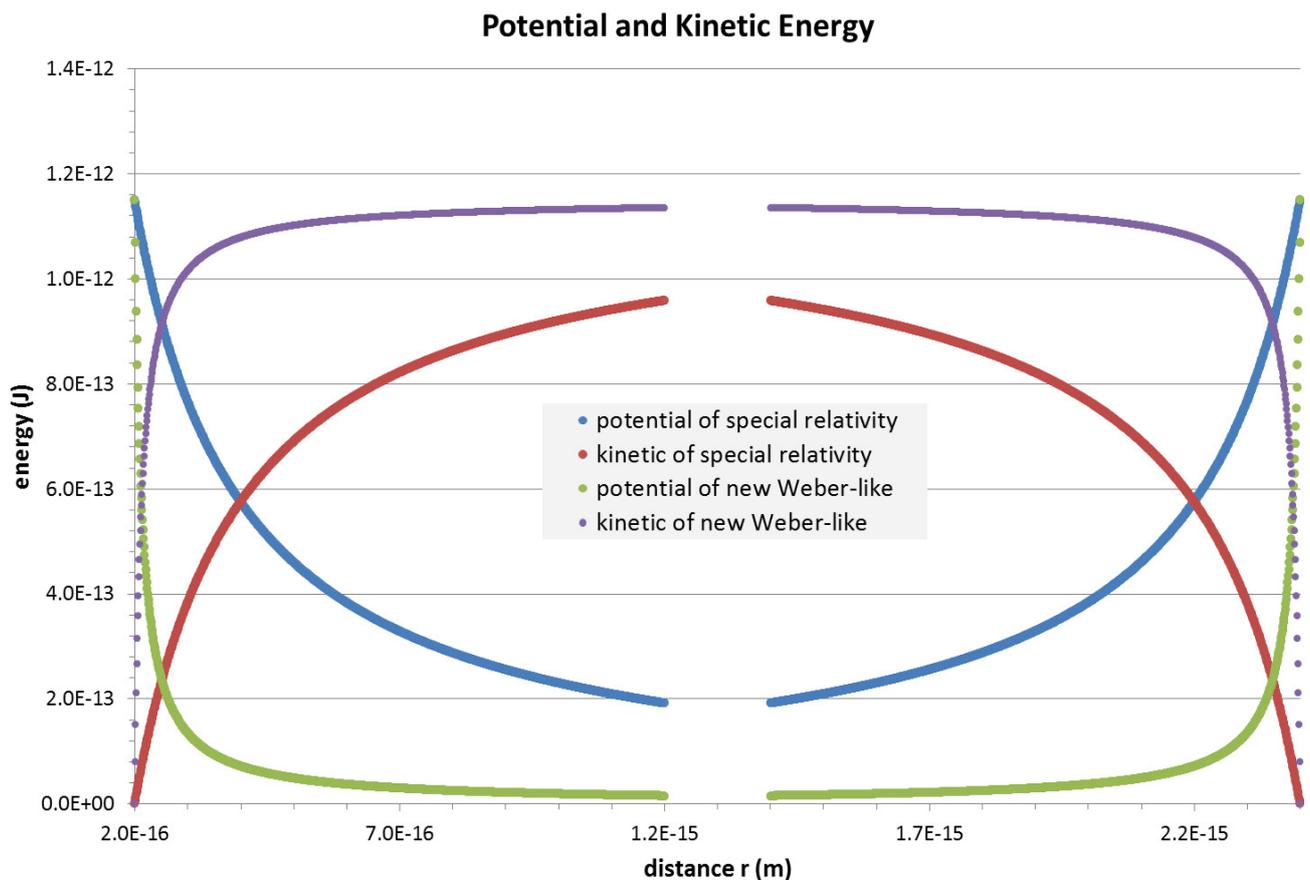


Figure 4: Illustration of particle energy by assuming particle q is a positron, and particle Q_1 and Q_2 have the same amount of charge as a positron. The new Weber-like potential energy and kinetic energy (equation 14) are compared with those from Special Relativity theory (equation 15).

However, the new Weber-like theory in this article indicates that the conversion is also dependent on velocity. The conversion amount between initial position r_0 and current position r is larger than that of Special Relativity theory (Figure 4).

Electrodynamic Force and Effective Mass

The electrodynamic force is the negative gradient of the potential energy. Let's again assume that particle Q is always at rest ($v_1 = 0$) (Figure 1). Then the electrodynamic force can be derived with equation (5):

$$F = -\frac{dU}{dr} = \frac{Qq}{4\pi\epsilon_0 r^2} \sqrt{1 - \frac{v_2^2}{c^2}} + \frac{Qq}{4\pi\epsilon_0 r} \frac{1}{c^2 \sqrt{1 - \frac{v_2^2}{c^2}}} \ddot{a}_2 \quad (16)$$

The force in the above equation is the same as that derived by Phipps [15]. Its low velocity approximation can become the Weber's electrodynamic force [10].

Let's use the force-acceleration relationship (equation 7) together with the above equation (16). Note that particle q has both original mass m_2 and additional mass m'_2 . We can get:

$$F = \frac{Qq}{4\pi\epsilon_0 r^2} \sqrt{1 - \frac{v_2^2}{c^2}} + \frac{Qq}{4\pi\epsilon_0 r} \frac{1}{c^2 \sqrt{1 - \frac{v_2^2}{c^2}}} \ddot{a}_2 = \frac{m_2 + m'_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} \ddot{a}_2 \quad (17)$$

Since Q is always at rest, we can get that $m'_1 = 0$ and $m'_2 = \frac{U_0}{c^2}$. After rearranging the above equation and using equation (15), we can get:

$$\frac{Qq}{4\pi\epsilon_0 r^2} \sqrt{1 - \frac{v_2^2}{c^2}} = m_2 \frac{c^2}{c^2 - v_2^2} \ddot{a}_2 \quad (18)$$

Thus, the effective mass (coefficient of acceleration in the right side of the above equation) is always positive in this specific case (one particle is moving and the other particle is always at rest).

Let's consider another scenario that particles Q and q are identical and particle velocities are much less than light speed (Figure 1). We can get $-v_1 = v_2$, $-\ddot{a}_1 = \ddot{a}_2$ and $m'_1 = m'_2 = \frac{U_0}{2c^2}$. The electrodynamic force can be derived with equation (5):

$$F = -\frac{dU}{dr} = \frac{Qq}{4\pi\epsilon_0 r^2} \sqrt{1 - \frac{(v_1 - v_2)^2}{c^2}} + \frac{Qq}{4\pi\epsilon_0 r} \frac{1}{c^2 \sqrt{1 - \frac{(v_1 - v_2)^2}{c^2}}} (\ddot{a}_2 - \ddot{a}_1) \quad (19)$$

Let's use the force-acceleration relationship (equation 7). We can get:

$$\frac{Qq}{4\pi\epsilon_0 r^2} \sqrt{1 - \frac{(v_1 - v_2)^2}{c^2}} + \frac{Qq}{4\pi\epsilon_0 r} \frac{1}{c^2 \sqrt{1 - \frac{(v_1 - v_2)^2}{c^2}}} (\ddot{a}_2 - \ddot{a}_1) = \frac{m_2 + m'_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} \ddot{a}_2 \quad (20)$$

After rearranging above equation, we can get:

$$\frac{Qq}{4\pi\epsilon_0 r^2} \sqrt{1 - \frac{(v_1 - v_2)^2}{c^2}} + \left(\frac{m_2 + m'_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} - \frac{Qq}{4\pi\epsilon_0 r} \frac{2}{c^2 \sqrt{1 - \frac{(v_1 - v_2)^2}{c^2}}} \right) \ddot{a}_2 \quad (21)$$

When the two particles are at rest, the effective mass (coefficient of acceleration in the right side of the above equation) can be simplified to:

$$m_{\text{eff}} = m_2 - \frac{3}{2c^2} \frac{Qq}{4\pi\epsilon_0 r_0} \quad (22)$$

If the two identical particles are very close to each other ($r_0 < \frac{3}{2c^2} \frac{Qq}{4\pi\epsilon_0 m_2}$), the effective mass becomes negative according to the above equation. Otherwise, the effective mass stays positive.

Discussions

In the new Weber-like potential energy (equation 2), we introduce a term $c^2 - v_1 \cdot v_2$ with respect to a laboratory rest frame. This potential energy expression works for cases when the relative velocity of two particles exceeds light speed, while the velocity of each particle stays below light speed. Because of this choice, we have to stick to the laboratory rest frame. If we switch to other reference frames, the new potential energy expression may not be valid. However, if the velocities are much less than light speed, the approximation of the new potential energy (equation 5) gains more flexibility, allowing coordinate systems other than the laboratory rest frame. It regains the relational properties the same way as Weber's electrodynamics does.

This article used a velocity dependent relationship between force and acceleration. It was proposed that mass (or inertia) is from the interactions with the bodies in the universe [22]. Using the Weber-like gravitational forces, Assis [23] derived a mass, which has a form independent to velocity. However, if a gravitational force similar to Phipps's electrodynamic force (or the force in this article) is used, the derived force-acceleration relationship could be velocity dependent and in a form as we postulated in this article.

To calculate the new Weber-like electrodynamic force, we need to know r , v_1 and v_2 , i.e. the current state of the two body system. However, to calculate particle acceleration, we also need to know the particle's mass, which include particle's original mass and the additional mass. The additional mass is related to the particle's energy, including potential energy (equation 12) and kinetic energy (equation 10). The mass-energy relation is the same as that introduced by Special Relativity theory [21]. To calculate the distribution of additional mass among particles, we need to use the laws of conservation of energy and momentum (equations 11). In the new theory, the total mass (original mass plus additional mass) has a conservation property, similar to energy conservation.

For the case of a charged particle inside a charged non-conducting shell, the new theory will not predict a negative mass (equation 18). Here, the shell is equivalent to the stationary particle, while the charged particle is equivalent to the moving particle. Since the potential energy between the particle and the shell gives an additional mass to the particle, the particle's mass will always be positive. This is different from the previous view using the Weber's electrodynamics and Newtonian kinetic energy [24]. However, when two particles are identical and both are free to move, we can still get negative mass (equation 22).

When one particle is far away from other particles, its kinetic energy in the new theory is in the same expression as that in Schrodinger's kinetic energy [14] or the kinetic energy of Special Relativity theory [21]. However, the kinetic energy in the new theory is higher than the kinetic energy of Special Relativity theory if there is potential energy between this particle and a nearby particle (Figure 3).

In this article, the equations are given in a one-dimensional sense. However, it can be extended to 3-dimensional by replacing velocity/acceleration in the equations with some products among velocity/acceleration vectors and unit distance vector between two particles. Similar conversion from radial velocity/acceleration to 3D velocity/acceleration vectors can be found in applications of Weber's electrodynamics [8].

Conclusions

We introduce a new theory of Weber-like potential energy, kinetic energy, and electrodynamic force, which are an extension of Weber's electrodynamics. At low velocity, this new theory can be simplified to Weber's electrodynamics. Thus it shall be able to explain electromagnetic phenomena as Weber's

electrodynamics does. On the other hand, the new theory overcomes the shortcoming of Weber's electrodynamics, such as negative mass of a charge inside a charged non-conducting shell. In the new theory, the particle velocity will not exceed light speed. At high velocity, the new theory is compatible with linear accelerator experiment result, which had only been explained by Special Relativity theory before.

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