



# Functional Roles of the Quantum Rayleigh Emission of Photons for Dielectric Integrated Waveguides - A Review

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## Abstract

Despite multiple classical outcomes arising from the quantum Rayleigh conversions of photons underlying the propagation of optical waves through dielectric media and the ensuing light-matter interactions, this quantum process has been largely ignored. Several of its outcomes are considered in this article from a physical perspective, e.g., inter-quadrature coupling of photons, phase-dependent amplification in optical directional couplers and related polarization rotation, phase-shifting of weak signals in the optically linear regime, location-dependent coupling coefficient for refractive index gratings, etc. A correct identification of these effects will enable useful design and operation of integrated photonic functional devices.

## Keywords

Quantum Rayleigh emission, Phase-sensitive amplification, Integrated photonic devices

## Introduction

The propagation of low intensity optical beams through dielectric materials is characterized as a passive process. Yet, an active physical interaction does exist which is capable of bringing about changes to the variable values of one optical wave which shares the medium with another optical wave of the same frequency. This interaction is induced by the quantum Rayleigh conversion of photons [1], underpinning the propagation itself by exciting and de-exciting electric dipoles in the dielectric medium, and leading to functional operations such as phase-sensitive amplification of photons, polarization rotation, spectral filtering, photonic noise reduction, etc. without requiring non-linear interactions [2,3].

Classically, Rayleigh scatterings [4] are interpreted as the dispersion of electromagnetic radiation by particles, e.g. neutral atoms, that have a radius smaller than approximately 1/10 the wavelength of the radiation. Optical energy associated with electromagnetic waves of high-frequency consists of an integer number of indivis-

ible amounts of energy, i.e., optical photons. The conservation of photon energy through absorption and emission is accompanied by local conservation of momentum mediated by the electronic cloud of the dipoles. Rayleigh scatterings can be found in any dielectric medium and dominate the attenuation of optical signals propagating along optical fibres [4].

However, alongside the classical Rayleigh scattering, fundamental quantum processes of Rayleigh conversion of photons occur which have been largely overlooked despite being documented in early textbooks [5,6] as well as underlying the process of propagation itself [1]. The quantum Rayleigh conversion of photons (QRCP) involves spontaneous and stimulated emissions of photons associated with absorption and emission of one photon per interaction and corresponds to the optically linear parametric conversion. This process underpins the propagation of photons in a dielectric medium by coupling photons from one quadrature of the optical wave into the next quadrature [1] in a continuous rotation on

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the phase space modifying the quadrature amplitude between zero and a maximum value. This process is distinct from the phase rotation on the phasor circle.

The quantum Rayleigh conversion of photons (QRCP) provides a physically meaningful explanation for the operation of the optical directional couplers, in contrast to the perturbation approach [7,8] based on the fictional splitting of the dielectric constant  $\epsilon(x, y, z) = \epsilon_b + \Delta\epsilon$  into a uniform background  $\epsilon_b$  and a perturbation  $\Delta\epsilon(x, y, z)$  which is supposed to generate a coupling polarization  $\Delta\mathbf{P} = \Delta\epsilon\mathbf{E}$ . Despite its physical contradictions [1] arising from the comprehensive wave equation, the coupling coefficient based on  $\Delta\epsilon$  is widely used, although, physically, the coupling coefficient is related to the total local value of the optical susceptibility  $\chi$  as opposed to the mathematical manipulation of  $\Delta\epsilon$ .

The QRCP can couple photons between two optical waves propagating simultaneously through a homogeneous medium, i.e., for which  $\Delta\epsilon = 0$ . The atomic electric dipoles absorb and emit photons, spontaneously, with the energized dipoles also serving as an excited medium available for stimulated emission. As a result, two optical waves of the same frequency, co-propagating through an optically linear and homogeneous dielectric medium, can exchange photons and undergo mutually induced phase-shifts. It is rather puzzling that the QRCP has been overlooked despite their ubiquitous presence [4].

The scatter pattern from a large number of photons  $N$  carried by an optical wavefront should be ergodically equivalent to the overall spatial pattern generated by a sequence of  $m$  wavefronts, each carrying  $N/m$  photons. In this context, the classical Rayleigh scattering - generated by the gradient of the dielectric constant, i.e.  $\nabla\epsilon$ , included in the wave equations - would be linked to the spontaneous emission while the coupling term  $\mathbf{P}_2 \cdot \mathbf{E}_1^*$  between one wave  $\mathbf{E}_1^*$  and the dipole polarization induced by a second wave  $\mathbf{P}_2 = \chi\mathbf{E}_2$ , and included in the complex Poynting vector [1] equation, would correspond to stimulated emission initiated by the spontaneous emission or another co-propagating wave of the same frequency.

This article brings together the analytical results of recent publications [1-3] and presents in Section 2 a physical approach based on the QRCP for a variety of integrated photonic devices such as phase-dependent amplification and polarization-dependent amplification, optical directional couplers, X-junctions, Y-junctions, polarisation rotation with spontaneous emission, etc. The role of spontaneously emitted photons by QRCP in initiating photon couplings is outlined in Section 3 along with polarization rotation of photons. A few physical aspects of QRCP are discussed in Section 4, which is followed by brief conclusions.

## Optically Linear Parametric Interactions in Optical Waveguides

The dipole polarizations involved in the interaction are linear, i.e.  $\mathbf{P} = \epsilon_0\chi\mathbf{E}$  where  $\chi(x, y, z)$  is the susceptibility of the medium, and in terms of the photon frequency notation,  $\mathbf{P}(\omega) = \epsilon_0\chi(\omega; \omega)\mathbf{E}(\omega)$  indicating a dipole polarization with one photon being absorbed and one photon emitted, both having the same energy [1].

An optical collimated beam, or a travelling radiation mode  $\mathbf{E}$ , characterized by a field amplitude  $E_0$ , an initial phase  $\varphi$ , an angular frequency  $\omega$ , a wave vector  $\mathbf{k}$  and field polarization unit  $\mathbf{e}$ , is represented, at time  $t$  and distance  $\mathbf{r} = (x, y, z)$  from origin, by the relations:

$$\mathbf{E}(\mathbf{k}, \varphi) = E_0(z) f(\mathbf{r}) e^{-i\varphi} e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \mathbf{e} \tag{1a}$$

$$\iint f^2(x, y, z) dx dy = 1 \tag{1b}$$

$$P(z) = 0.5 \epsilon_0 n c E_0^2(z) \tag{1c}$$

The field distribution  $\mathbf{E}$  is given in terms of the peak amplitude  $E_0$  and the spatial distribution  $f(x, y, z)$  which has units of  $m^{-1}$  and is normalized across any  $(x, y)$ -plane in eq. (1b) so that  $P(z)$  represents the total average power crossing that surface. Additionally,  $\epsilon_0$  is the permittivity of free space,  $n$  is the refractive index of the medium, and  $c$  is the speed of light in vacuum. The polarization unit vector for linearly polarized light is denoted by  $\mathbf{e}$ .

The optical field  $\mathbf{E}(\mathbf{r})$  at location  $\mathbf{r}$  interacts instantaneously ( $10^{-14}$  s) with the local susceptibility  $\chi$  in its entirety. An oscillating dipole polarization  $\mathbf{P}_2$  can radiate into mode  $\mathbf{k}_1$  and a mutual interaction emerges from the coupling term  $\mathbf{P}_2 \cdot \mathbf{E}_1^*$  incorporated in the Poynting theorem. The differential, local and temporal, Poynting theorem [1] describing the optical flow of energy has the following form, with the asterisk denoting the complex conjugate of the variables of the displacement vector  $\mathbf{D}$ , the magnetic field  $\mathbf{H}$  and the corresponding magnetic induction  $\mathbf{B}$ :

$$\nabla \cdot \mathbf{P}_1 = -i\omega \mathbf{D}_1 \cdot \mathbf{E}_1^* - i\omega \mathbf{B}_1 \cdot \mathbf{H}_1^* - i\omega \mathbf{P}_2 \cdot \mathbf{E}_1^* \tag{2}$$

Where  $\mathbf{P}_1 = \mathbf{E}_1 \times \mathbf{H}_1^*$  is the Poynting vector parallel to the wave vector  $\mathbf{k}_1$ , and the vectors  $\mathbf{k}$ ,  $\mathbf{E}$  and  $\mathbf{H}$  are perpendicular to each other for the same radiation mode. We align  $\mathbf{k}_1$  to be parallel to the z-axis in a waveguide. It is Eq. (2) of the Poynting vector which accurately describes interactions between two optical waves [1], rather than the Helmholtz wave equation [8].

The equations of motion [3] for the coupling between two optical waves identified by the subscripts  $a$  and  $b$  have the following forms for the powers  $P$ , phases  $\varphi$ , the gain coefficient  $g$ , and the coupling coefficient  $\kappa$ :

$$d P_a/dz = g_a(z, \theta_b a) P_a - \alpha P_a \tag{3a}$$

$$g_a(z, \theta_b a) = -2 \kappa r_a(z) \sin\theta_b a(z) \tag{3b}$$

$$r_{a,b}^2(z) = \frac{P_{b;a}}{P_{a;b}} \quad (3c)$$

$$\theta_{ba}(z) = [\beta_b - \beta_a] \cdot z + \varphi_b - \varphi_a = \Delta\beta \cdot z + \Delta\varphi \quad (3d)$$

$$\frac{d}{dz} \theta_{ba}(z) = \Delta\beta + [r_b r_a] \cos \theta_{ba}(z) \quad (3e)$$

$$\varphi_a(z) = \varphi_a(0) + \int_0^z r_a(s) \cos \theta_{ba}(s) ds \quad (3f)$$

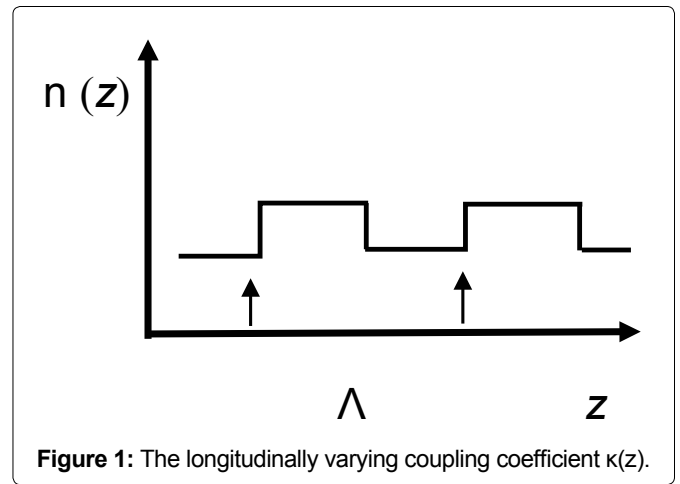
$$\kappa = \frac{k_o}{2n} \iint dx dy \chi^{(1)} f_a f_b \mathbf{e}_a \cdot \mathbf{e}_b \quad (3g)$$

Where the loss factor is denoted by  $\alpha$ , the propagation constant being  $\beta_a \mathbf{u}_z = k_o n \mathbf{u}_z$ , with  $\mathbf{u}_z$  being a unit vector in the  $z$ -direction, and  $\theta = (\beta_b - \beta_a) \cdot z + \varphi_2 - \varphi_1$  is the phase difference between the two fields.

Eqs. (3a-g) describe the physical process of quantum Rayleigh conversion of photons [3], leading to useful aspects of these interactions. A non-vanishing coupling coefficient  $\kappa$  in an optical waveguide requires that the lateral field distributions  $f(x, y, z)$ , for a uniform or symmetric susceptibility, should both be either symmetric or asymmetric. In this case, coupling of photons between two modes can take place, being limited by their phase-mismatch. The direction of power coupling tends to equalize the powers in the two modes as the phase shifts bring about this outcome [3]. The gain coefficient is phase- and polarisation -sensitive enabling selective amplification. Overall, it is possible to control the properties of a signal wave by adjusting the input values of a low-power pump wave because of the strong value of the optically linear susceptibility  $\chi$ . By using the same optical frequency for an entire photonic circuit made up of dielectric waveguides, it should be possible to reduce the complexity associated with multiple waves and types of materials. Electro-optic waveguides provide connections between the electrical signals and optical ones [9].

From Eqs. (3) we see that power conversion in an optical directional coupler composed of two optical waveguides will take place in the cladding medium separating the two waveguides [1]. The gain coefficient is phase-dependent and the relative phase (or phase-mismatch) can be controlled by adjusting the power ratio  $r$  between the pump and the signal waves. For converging or diverging Y-junctions, the coupling coefficient would have polarisation vectors projected onto each other. The coupling coefficient  $\kappa$  includes contributions from both the transverse and longitudinal components of the optical field [1], unlike the conventional derivation [10].

The case of a refractive index grating - see Figure 1 - would be of particular interest in order to couple power between two waves of different propagation constants,



either co-propagating or counter-propagating. The susceptibility (or the refractive index  $n^2 = \epsilon/\epsilon_0 = 1 + \chi$ ) becomes periodic  $\chi(z) = \chi_0 + \Delta\chi(z)$  leading to a  $z$ -dependent coupling coefficient  $\kappa(z)$ . The coupling gain is maximized for the overlap in Eqs. (3b) and (3g) of  $\Delta g(z, \theta) \propto \Delta\chi \cdot \sin(2\pi z/\Lambda) \cdot \sin(\Delta\beta \cdot z + \Delta\varphi)$  resulting in the phase-matching condition  $2\pi/\Lambda = \Delta\beta + d(\Delta\varphi)/dz$  between the periodicity of  $\chi$  and the relative phase  $\theta$ . The last equality along with Eq. (3f) indicate the existence of a built-in mechanism for phase-matching between a strong pump and a weak signal through the parametrically induced phase shift of Eq. (3e). A rectangular periodic perturbation of the susceptibility - see Figure 1 - results in step-by-step amplification as the coupling in the first half of the period  $\Lambda$ , from one wave to the other is stronger than the opposite coupling over the second half of the period where  $\Delta\chi = 0$ . We point out that the coupling coefficient varies with distance and it is not the Fourier transform of the refractive index modulation function as suggested in [10,11].

From Eqs. (3f) an optically linear parametric phase-pulling effect emerges [3], which shifts the phase of a weak signal towards a  $(-\pi)/2$  difference from the phase of strong pumps. Inter-quadrature coupling between the components of one single wave through quadrature amplification of spontaneous photons accompanies the propagation, leading to the rotating power distribution of a propagating field which is described by [1, Eq. (5b)]:

$$E(z, t) = E_0 [\cos(\kappa z) \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) - \sin(\kappa z) \sin(\omega t - \mathbf{k} \cdot \mathbf{r})].$$

Equally, two optical waves co-propagating through a dielectric medium will couple photons from one wave to the other depending on the relative phase between the two waves. This process, repeatedly, will eliminate optical waves whose phases diverge substantially from the phase of the surviving wave which will dominate the output and preserve the coherence of the optical beam

despite light-matter interactions.

## Spontaneous Emission and Polarization Rotation

A critical difference between the classical optics and the quantum formulation is the spontaneous emission. While classically an optical field can start with a zero value, quantum optically the growth of an optical beam is initiated by spontaneously emitted photons. The probability of emitting a photon with the momentum  $\mathbf{k}$  and polarization  $\mu$  is related to the decay rate  $\gamma_s$ [1/s] of the excited dipole in free space evaluated as [12]:

$$\gamma_s^{(\text{free})}(\mathbf{k}, \mu, \omega) = \frac{\omega^3}{\hbar c} \left( \frac{\mathbf{d} \cdot \mathbf{e}_{\mathbf{k}\mu}}{4\pi} \right)^2 \quad (4)$$

With  $\mathbf{d}$  denoting the electric dipole moment vector, and  $\mathbf{e}_{\mathbf{k}\mu}$  the polarization unit vector of the emitted photon, and which is perpendicular to the direction of propagation  $\mathbf{k}$ .

In a dielectric material of constant  $\epsilon$ , the decay rate is modified, so that [12]:

$$\gamma_s(\mathbf{k}, \mu, \omega) = \frac{9 \epsilon^{5/2}}{(2\epsilon + 1)^2} \gamma_s^{(\text{free})}(\mathbf{k}, \mu, \omega) \quad (5)$$

But its angular distribution is the same as in free space.

The case of forward propagation, in the  $z$  - direction, of the polarization-independent spontaneous emission  $P_{sp}$  for a pump power  $P_p$  was evaluated in [1, App. B] as  $P_{sp} = \chi^2 h P_p \Omega \cdot \Delta z / (\pi n \lambda^4)$  where  $h$  is Planck's constant,  $\lambda$  is the wavelength, and  $\Omega$  is the solid angle, centered on the fiber's axis, into which photons are emitted and are likely to be captured by the guided mode. This solid angle has the geometrical shape of a cone and its apex half-angle is  $\alpha = \arccos(\beta/k_{\text{core}}) = \arccos(n_{\text{eff}}/n_{\text{core}})$  so that  $\Omega = \pi \alpha^2$ ,  $k_{\text{core}}$  and  $n_{\text{core}}$  being, respectively, the wavenumber and the refractive index of the core, and  $n_{\text{eff}}$  being the effective index of the mode. For  $\lambda = 1.55 \times 10^{-6}$  m,  $\Omega = 2 \times 10^{-2}$  sr, and  $\Delta z = 2 \times 10^{-6}$  m, one calculates  $P_{sp} = 2 \times 10^{-19} P_{\text{pump}}$ .

Correspondingly, after setting  $N = P/(\hbar \cdot \omega)$ , the number of pump photons emitted spontaneously into a particular solid angle  $\Omega$  over a distance  $\Delta z$  is given by [1]:

$$N_{sp}(\Delta z) = \frac{\chi^2 h N_{\text{pump}} \Omega \cdot \Delta z}{\pi n \lambda^4} \quad (6)$$

The accumulated number of spontaneously emitted photons is found by combining Eqs. (4 - 6), leading to a directional distribution of:

$$N_{sp}(\Delta z, \theta_{em}) = N_{sp}(\Delta z) (\cos \theta_{em})^2 \quad (7)$$

With  $\theta_{em}$  the emission angle between the dipole  $\mathbf{d}$  and the polarization unit vector  $\mathbf{e}_{\mathbf{k}\mu}$  of the photons.

Spontaneously emitted photons with  $\pm \theta_{em}$  polariza-

tion angle relative to the  $x$  direction, will be amplified through the coupling coefficient  $\kappa \propto \cos \theta_{em}$  of eq. (3 g). With  $\mathbf{e} \cdot \mathbf{x} = \cos \theta_{em}$  and  $\mathbf{x} \cdot \mathbf{y} = 0$ , a non-vanishing value along the  $y$  -polarization is obtained by blocking off either  $+\theta$  or  $-\theta$  polarized photons, as the two polarization states would cancel out each other in the  $y$  - direction.

## Physical Aspects of the Coupled Wave Formalism

Despite its physical flaws and contradictions [1], the coupled-mode theory (CMT) based on the perturbation of the refractive index in the cladding between two parallel waveguides has been used for decades [8]. The CMT was disseminated in the 1970s in a series of textbooks which made the theory practically sacrosanct despite it being physically impossible [1]. Yet, a physically meaningful and correct identification of processes underpinning the description of photon coupling in dielectric media is crucial in order to open up new practical ways of designing, fabricating, and operating integrated photonic devices [2,3,9].

An optically linear parametric (OLP) interaction consists of an electric dipole absorbing one photon and emitting one photon of the same frequency or energy, e.g. the elastic or Rayleigh scatterings [6,7]. This quantum process is localized, can take place in both homogeneous and inhomogeneous dielectric media, and is described macroscopically, or classically, by means of the interaction term included in the Poynting theorem of the flow of energy.

Quantum electronically, an oscillating electric dipole polarization can be the source of spontaneous and stimulated emissions of photons [5,6]. Consequently, coupling of photons between two arbitrary waves interacting simultaneously with the same dielectric medium can take place with one beam of photons exciting the electric dipole polarization and the other beam de-exciting the dipoles and gaining power through stimulated emission. The direction of coupling will depend on the relative phase between the waves. These physical processes require that Maxwell's curl  $\mathbf{H}$  equation of *each* wave be driven by the *total* electric dipole polarization available in the medium [1].

Furthermore, the comprehensive wave equations incorporating the gradient of the dielectric constant  $\nabla \epsilon$  would cause the tail end of the evanescent field of one waveguide to be scattered by the existence of the second waveguide [1].

It is the effect of stimulated emission which is capable of causing photons to propagate with the same wave vector as the stimulating field instead of being spontaneously emitted or classically scattered. The gain providing

stimulated emission is always the source of spontaneous emission. The spontaneous emission, which is a feature of the quantum wave-dipole interactions, is dependent on the optical susceptibility and the level of the optical pump and is distinct from the zero-point fluctuations of the electromagnetic field.

Additionally, photon coupling in homogeneous waveguides will rule out mode orthogonality as a standard feature for optical waveguides. The property of mode orthogonality was predicated on there being no coupling in a homogeneous dielectric medium.

## Conclusions

The existence of quantum Rayleigh conversions of photons in dielectric optical waveguides rebuts the common view that these waveguides are passive devices. Low levels of optical powers can be used with the optically linear susceptibility to couple photons through spontaneous emission and stimulated processes of phase-sensitive amplification accompanied by phase shifts. These low-power mechanisms should enable compact and efficient integrated photonic devices for a range of applications based on light-controlled interactions.

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