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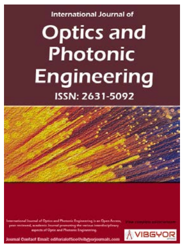

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Beam Shaping Optical Lens Designs for Diffraction-Free Bessel Beams

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Abstract

Beam shaping technique is applied to design optical lenses for transforming uniform beams to diffraction-free Bessel Beams. One-lens and two-lens refracting system will be demonstrated where the lens' surfaces equations can be easily derived. The designed lenses take the input uniform beam distribution and confine it to be within the main lobe of the Bessel beam and avoid its zeros crossings. Various design parameters such as the power of the beam and the optical system length will be discussed. It will be demonstrated that the two-element design will produce smaller system length than the one-element design.

Keywords

Beam shaping methods, One-element and two-element refracting system, Lens design

Introduction

The fundamental mode (TEM_{00}) of the cavity of Gaussian laser beam is the most common beam shape used in optical processing. In particular, for materials processing [1-8] the low divergence of the Gaussian beam provides a small focused spot. It can be shown that a focal spot size as small as 1 μm can be obtained using a lens with a given numerical aperture NA [3]. The small spot size is very attractive for microfabrication applications. Further, for laser materials processing, the depth of field (DOF) or the distance (from the beam waist over which the focal beam size and the peak intensity keep their values) is very important factor. The DOF of Gaussian beam is too short for many industrially important processing applications [1] due to transverse spreading. As the use of laser in materials processing increases, researchers increase their efforts in exploring other non-Gaussian laser beams (due to its short, micron-sized Rayleigh range), such as the non-diffracting Bessel beams [9], which have an accurate laser beam focusing over a good distance range. This comes at the expense of reducing the amount of energy available in the process where Gaussian beam has an advantage over any other beam shape.

Bessel beams, in contrast to Gaussian beams, have extended DOF with a minimal spreading in the central lobe during wave propagation. This non-diffracting property attracts researchers to attempt using Bessel beams in many new applications. The researchers' efforts appear, recently, in much usage of micro and femtosecond lasers to process a broad variety of materials such as metals,

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glass, semiconductors, biological samples, etc with very high degree of precision and reproducibility, and with minimized laser induced damage. Bessel beams can be generated by many methods [10-15]. Lasers with high output powers are required for a number of applications, such as for material processing (welding, cutting, drilling, soldering, marking, surface modification) [7,16], where many of these beams are generated in unstable resonator. For instance, using a ring-type reflective mirror inside the resonator results in an output annular beam profile.

On the other hand, beam shaping techniques were proposed to convert laser beam profiles to other beams profiles [17,18]. Refractive optical system technique is among many successful and efficient methods that can achieve this conversion. This technique relies on geometric optics for designing laser beam shapers. The designed optical element functions as a field mapping of the input beam distribution to provide a desired output beam profile. In the past one-element and two-element refracting systems were reported to transform annular-Gaussian to Bessel beam [19-23]. Recently, we have proposed the design of such refractive systems for generating various other laser beams profiles [24,25] such as:

(i) Annular-uniform-to-uniform;

(ii) Annular-Gaussian-to-uniform;

(iii) Gaussian-to-uniform;

(iv) Gaussian to Annular-uniform;

(v) Annular-Gaussian to Annular-uniform;

(vi) Annular-Gaussian to Gaussian;

In this paper, we further extend the previous works to design refractive systems to transform uniform circular beam to diffraction-free circular Bessel beams using one-element and two-element lenses. A procedure to derive the mathematical expressions of the input and output lens surfaces is provided for beams transformations from uniform to Bessel. The designed lenses take the input beam distribution and confine it to be within the main lobe of the Bessel beam and avoid its zeros crossings. It will be shown that for a given system specified with certain dimensions and power concentration, then the design parameters can be

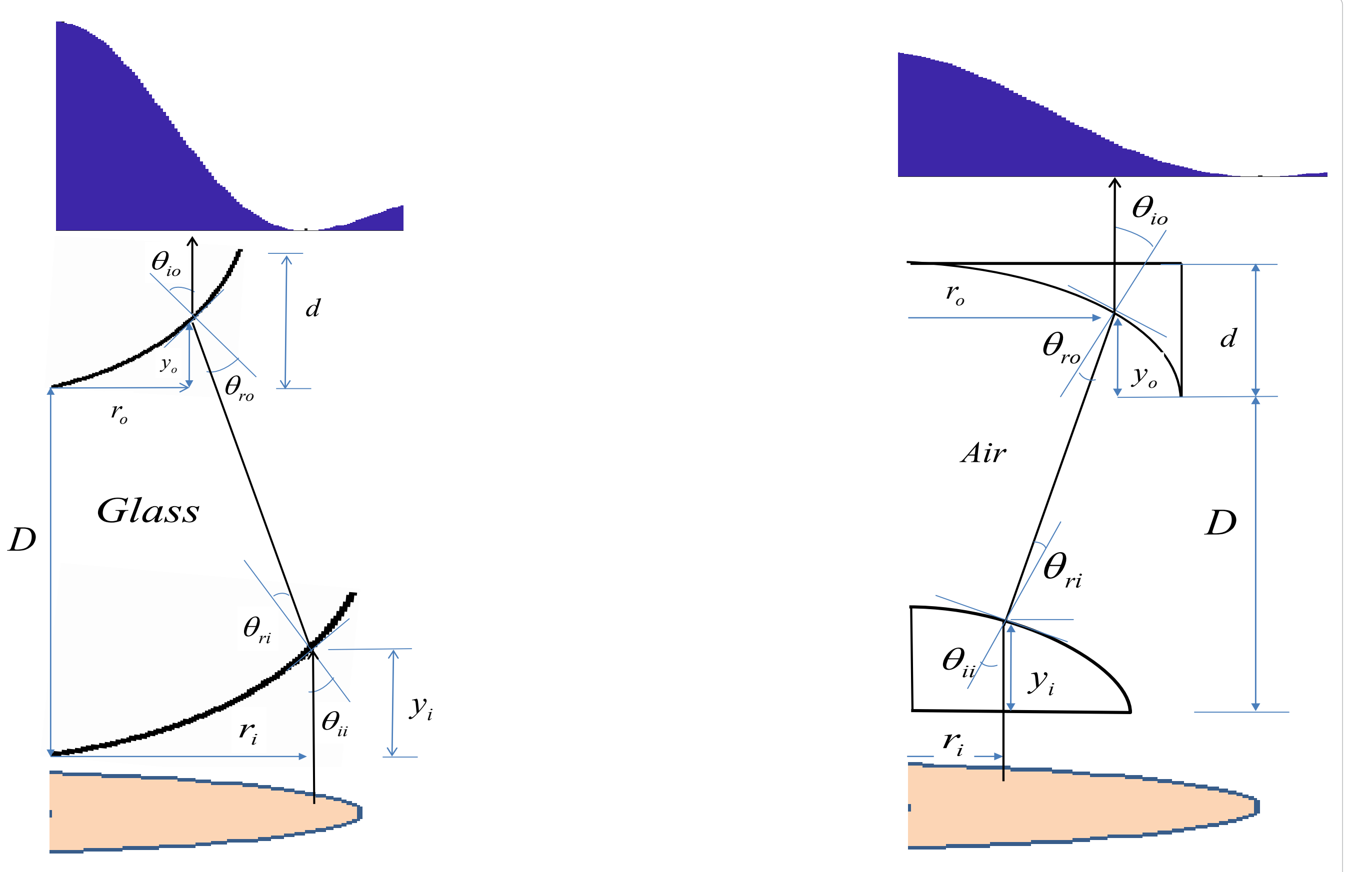

Figure 1: The geometric set-up of the refraction system beam transformation: a) One-element design and b) Two-element design.

changed to meet these specifications. Further, the designed surfaces of the lenses are smooth enough to be fabricated. It will be demonstrated that the two-element design will produce smaller system length than the one-element design.

Design Considerations

Figure 1a and Figure 1b show schematics of half of the proposed axially symmetric refracting system for uniform to Bessel beam transformation for one-element and two-element, respectively. The lens is made from glass with index of refraction n = 1.5172. D is the distance between the input and the output horizontal reference planes. Starting with an incident ray that hits the first surface of the lens with an angle ϑ_{ii}, it will be refracted by an angle ϑ_{ri}. The refracted ray reaches the second lens surface with an angle ϑ_{ro} and then it is refracted by an angle ϑ_{io}. Note the parallelism between the incident and the exited ray. The lens surfaces are represented by $y_i(r_i)$ and $y_o(r_o)$ functions of the radial distances r_i and r_o of the input and output surfaces, respectively.

Beam shaping method design imposes few conditions that must be met to realize beam conversion such as: a) All input rays that enter and leave the lens must travel the same optical path length; b) All input rays that enter and leave the lens must be parallel to each other's; and c) The ratio of the input beam power to the output beam power must equal to a constant.

For the one-element design, the first two design conditions are translated into the following equations:

$$y_i + n\sqrt{(r_i - r_o)^2 + (D - y_i + y_o)^2} + d - y_o = f \qquad (1)$$

$$tan(\theta_{ii} - \theta_{ri}) = tan(\theta_{io} - \theta_{ro}) = (r_i - r_o)/(D - y_i + y_o) \qquad (2)$$

Where f is a constant and $f' = f - d - D$. Further, the input and the output rays parallelism dictates that both the input and the output surfaces slope must be equal to each other, i.e.

$$dy_i / dr_i = dy_o / dr = tan\theta_{ii} = tan\theta_{io} \qquad (3)$$

Furthermore, using Snell's law at the input lens ($\sin\vartheta_{ii} = n\sin\vartheta_{ri}$) along with trigonometric identities, one can obtain:

$$f' = (r_i - r_o)\left[\frac{n - cos(\theta_{ii} - \theta_{ri})}{sin(\theta_{ii} - \theta_{ri})}\right] \qquad (4)$$

And

$$dy_i / dr_i = dy_o / dr_o = n\{(f'/(r_i - r_o))^2 - n^2 + 1\}^{-1/2} \qquad (5)$$

Note that Eq. (5) imposes a minimum value on the length f' to avoid the singularity. On the other hand, similarly, for the two-element design, we can obtain the following equations:

$$ny_i + \sqrt{(r_i - r_o)^2 + (D - y_i + y_o)^2} + n(d - y_o) = c \qquad (6)$$

$$c' = (r_i - r_o)\left[\frac{sec(\theta_{ri} - \theta_{ii}) - n}{tan(\theta_{ri} - \theta_{ii})}\right] \qquad (7)$$

$$dy_i / dr_i = dy_o / dr_o = -tan\theta_{ii} = -tan\theta_{io} \qquad (8)$$

$$dy_i / dr_i = dy_o / dr_o = -\{(c'/(r_i - r_o))^2 + n^2 - 1\}^{-1/2} \qquad (9)$$

Where c is a constant and $c' = c - nd - D$.

Laser Beam Transformations

The first step in the procedure for designing the refracting lens starts with establishing a relationship between r_i and r_o the radial distances of the lenses' surfaces. The above-mentioned third condition of beam shaping can be used in this regard. This condition is translated into a relationship between the cross-sectional areas of the uniform and the Bessel beams:

$$\pi r_i^2 = k^2 \int_0^{r_o} 2\pi\rho J_0^2(\alpha\rho)\, d\rho \qquad (10)$$

Where J_0 is the zero-order Bessel function of first kind, r_i is the radius of the input circular beam, r_o is the radius of the output Bessel beam, k is a constant that defines the amplitude, and α is a parameter that sets the width or diameter of the main lobe of the Bessel beam as shown in Figure 2. The diameter of the central lobe is a main parameter that may control the Rayleigh range [3] for Bessel beam and the power in the central lobe. Eq. (10) leads to the following relationship:

$$r_i^2 = (r_o k)^2 [J_0^2(\alpha r_o) + J_1^2(\alpha r_o)] \qquad (11)$$

Eq. (11) expresses the relationship between four parameters in the lens design, i.e., r_i, r_o, k, and α. The parameter α sets the first zeros in the Bessel beams that are approximately 4, 2.4, and 1.2 for α = 0.6, 1, and 2, respectively. The designer may select a specific beam spot size r_o for a particular beam and obtain the size of the input beam r_i. For instance, for Bessel beam with α = 0.6, the input power will be delivered to a spot size of r_o = 2 (50% of central lob width). Thus, for α = 0.6, r_o = 2 and for a selected value for k (say k = 1), we can easily determine the required value of the input beam r_i =

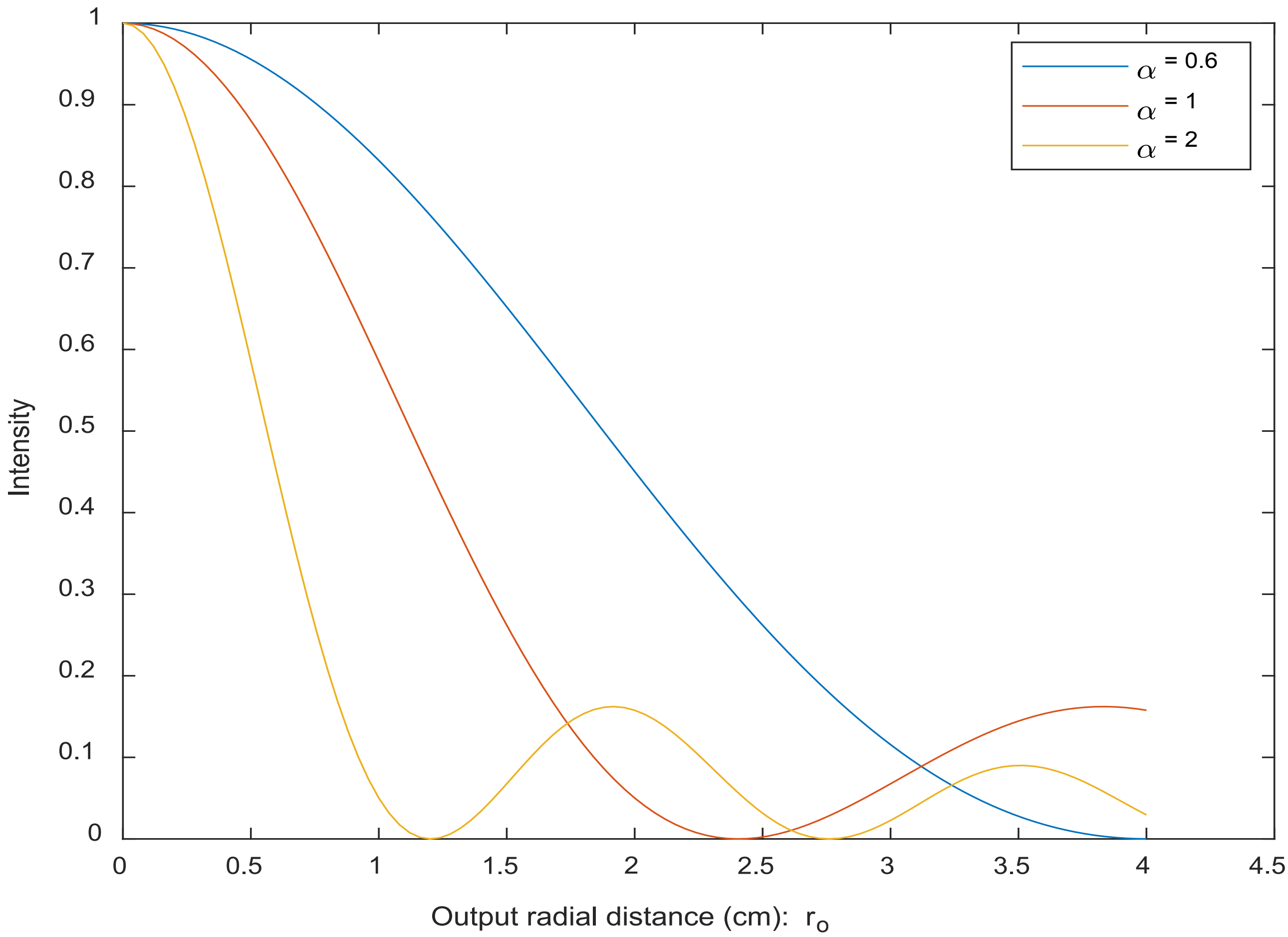

Figure 2: Intensity of zero-order Bessel functions for different α.

1.68 and the minimum value of f' = 0.37. Similarly, for 95% of the central lobe (r_o = 3.8), r_i is found to be 2.08.

On the other hand, Eq. (11) shows that k has effects on the value of r_i. At the same time, the value of f' in Eq. (4) depends on r_i. Further, Eq. (5) depends on both f' and r_i. As a result, varying k will eventually vary the minimum value of the length of the system and the lens input and output surfaces. For example, for $\alpha = 0.6$, $r_o = 2$, $k = 0.5$; then $r_i = 0.84$ and the minimum value of $f' = 1.33$.

The second step in the beam shaping design procedure is to specify a value of the system length (f', c', or D), which determine the numerical values for the surfaces slope dy_i/dr_i and dy_o/dr_o from Eqs. (5) and (9). The third step in the design procedure is to use a curve-fitting routine to determine mathematical expressions for these slopes from their numerical values. Finally, by integrating the mathematical expressions of these slopes, we can obtain the mathematical expressions of the surfaces $y_i(r_i)$ and $y_o(r_o)$. Note that in the design procedure, selecting design parameters should take into consideration that the system length to be as small as possible and the radii of curvature, $\frac{[1+(dy/dr)^2]^{3/2}}{d^2y/dr^2}$, for the lens surfaces to be large. In the following subsections, we will present designs for one- and two-element system for different parameters values of k, α, and (f' or c'). These parameters will change the shape of the lens and the size of the designed system.

One-element designs

For the one-element design, we will present four designs to demonstrate the effects of parameters k and α. For the first two designs, we choose f' = 1.35 cm (D = 2.61), k = 1, r_o = 2 (50% of central lob width), for α = 1 and α = 0.6, as shown in Figure 3 and Figure 4. In these figures, when α decreases, then the input radial distance r_i increases from r_i = 1.24 to r_i = 1.68. Further, for α = 1, it is found that the minimum value for f' is 0.87 which corresponds

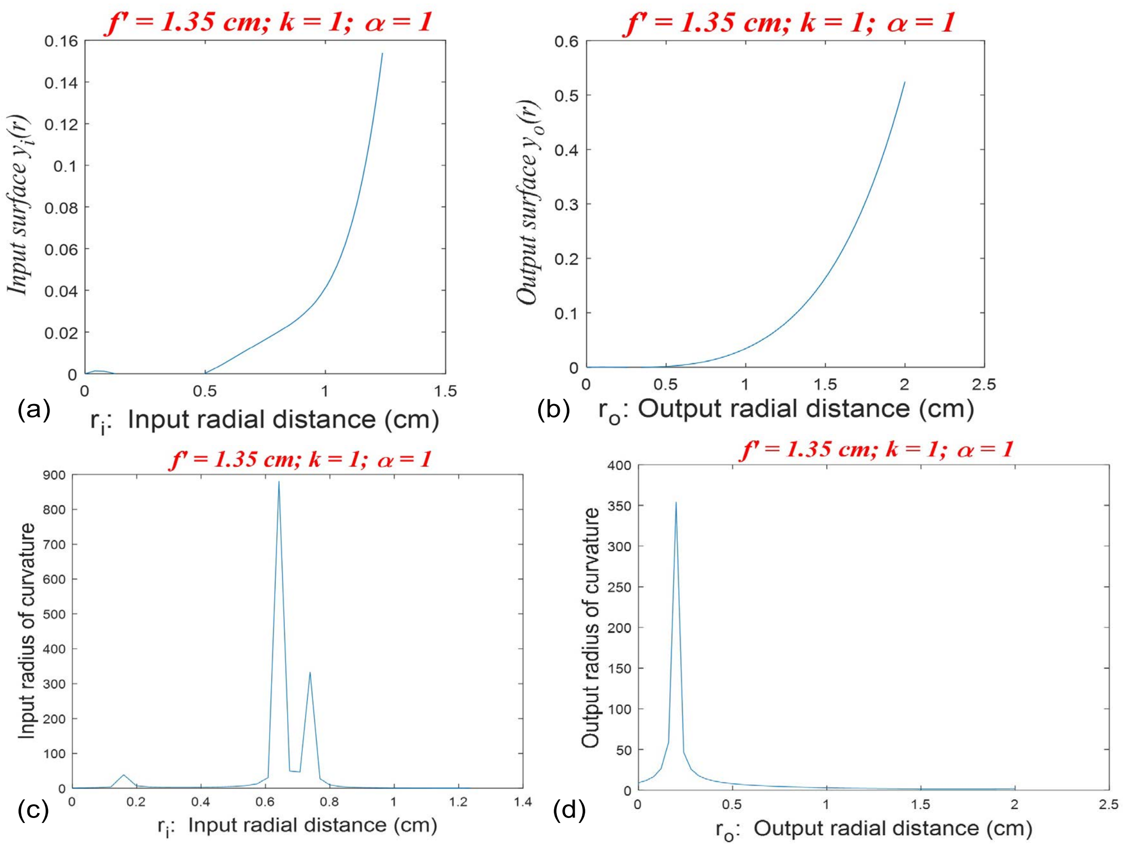

Figure 3: One-element design for uniform to Bessel for f' = 1.35, D = 2.61, k = 1, α = 1: a) and b) The input and the output surfaces; c) and d) The input and the output radius of curvature of the surfaces.

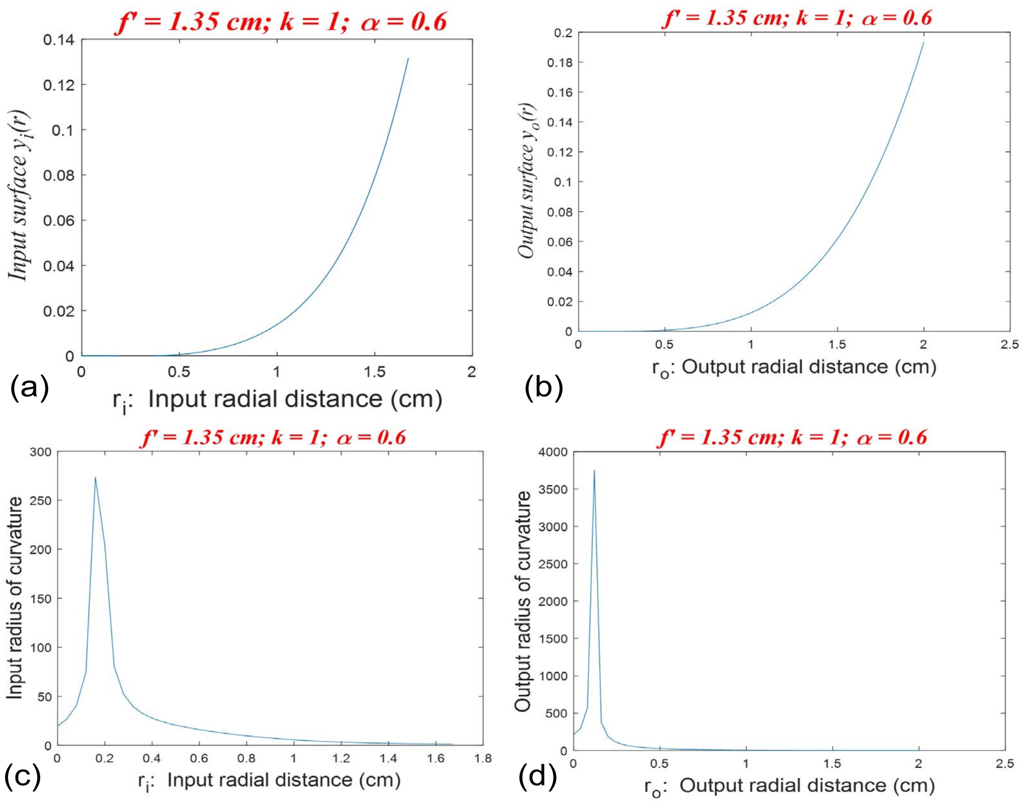

Figure 4: One-element design for uniform to Bessel for f' = 1.35, D = 2.61, k = 1, α = 0.6: a) and b) The input and the output surfaces; c) and d) The input and the output radius of curvature of the surfaces.

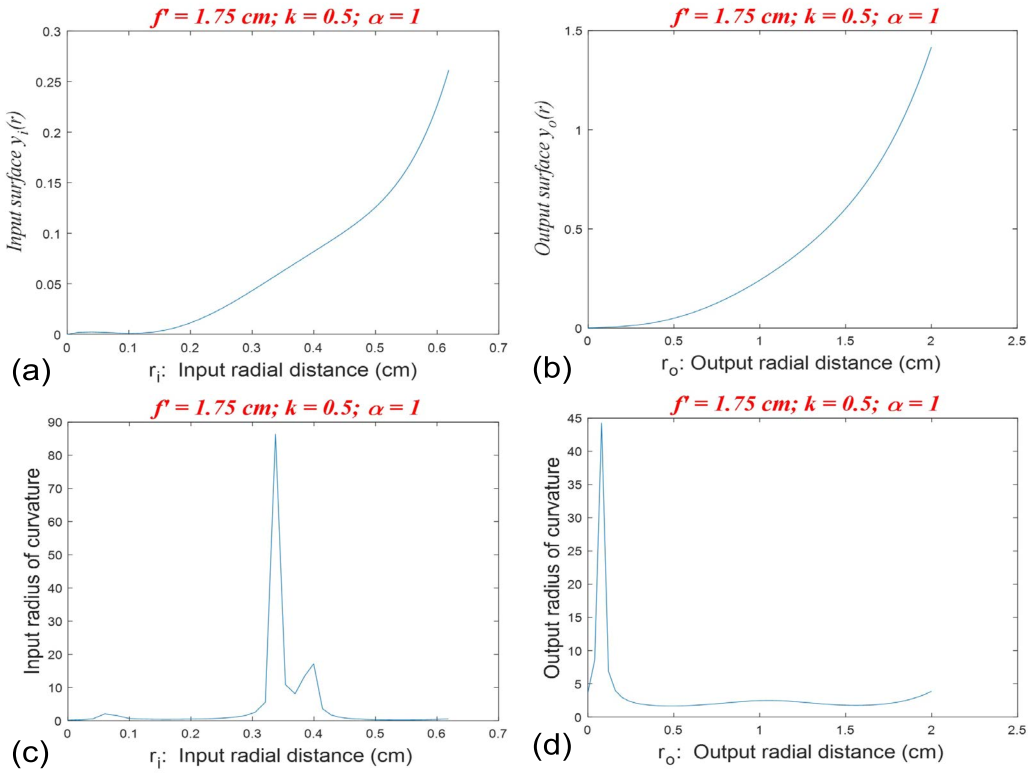

Figure 5: One-element design for uniform to Bessel for *f'* = 1.35, *D* = 2.61, *k* = 0.6, *α* = 1: a) and b) The input and the output surfaces; c) and d) The input and the output radius of curvature of the surfaces.

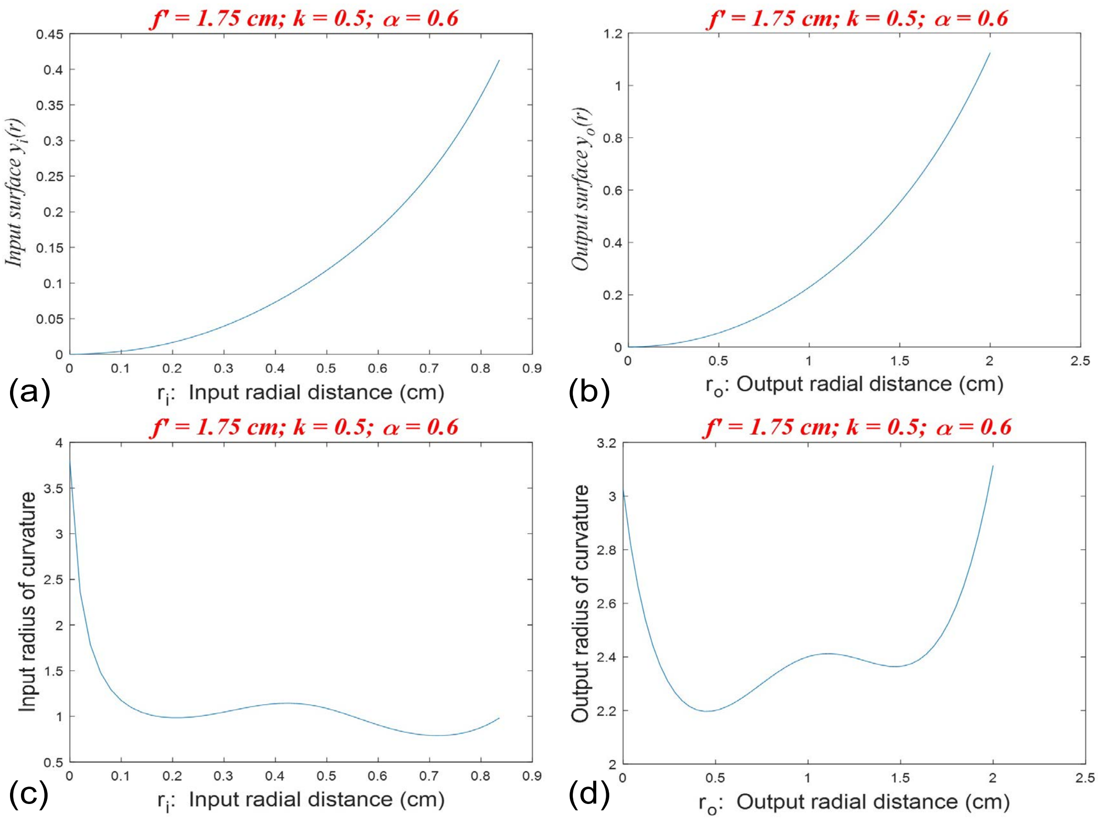

Figure 6: One-element design for uniform to Bessel for *f'* = 1.35, *D* = 2.61, *k* = 0.5, *α* = 0.6: a) and b) The input and the output surfaces; c) and d) The input and the output radius of curvature of the surfaces.

Table 1: Various design parameters for one-element lens design.

		α = 0.6			α = 1			α = 2		
	k	0.5	1	2	0.5	1	2	0.5	1	2
(50%) of central lobe	r_o	2			1.2			0.6		
	r_i	0.84	1.68	3.36	0.5	1.0	2.0	0.25	0.50	1.0
(95%) of central lobe	r_o	3.8			2.28			1.14		
	r_i	1.04	2.08	4.16	0.62	1.24	2.48	0.31	0.62	1.24

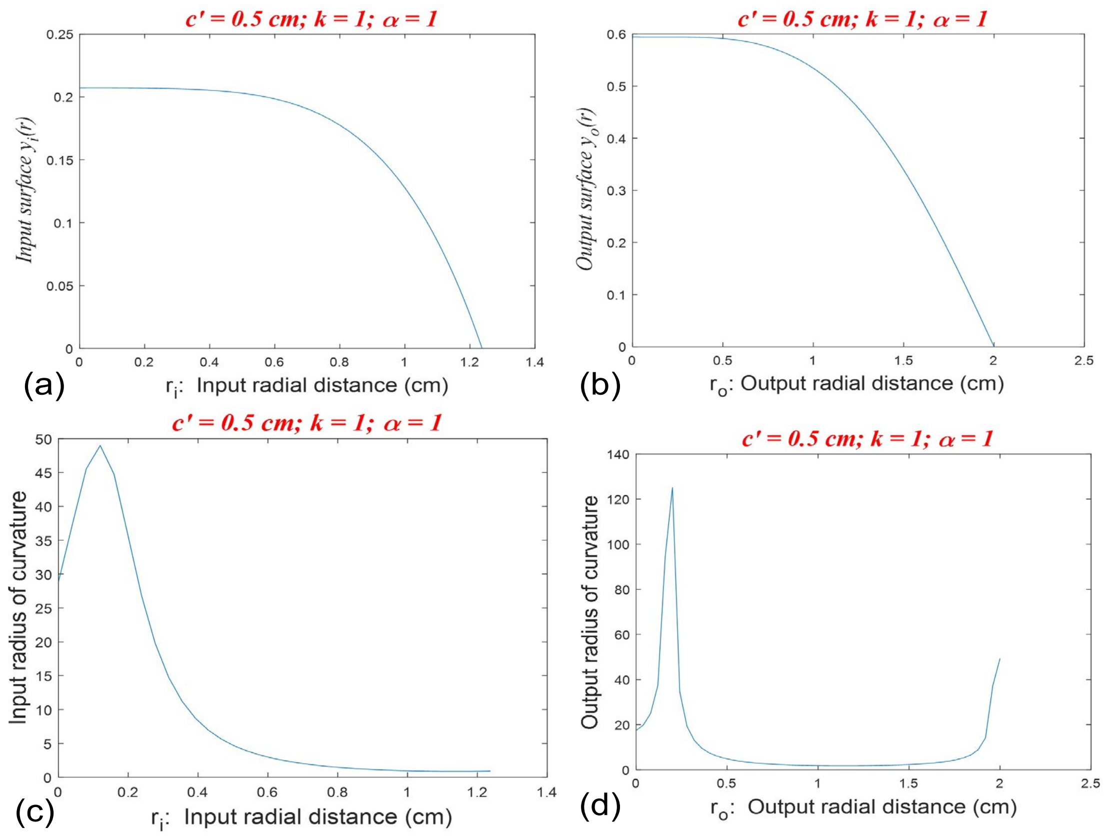

Figure 7: Two-element design for uniform to Bessel for c' = 0.5, D = 0.96, k = 1, α = 1: a) and b) The input and the output surfaces; c) and d) The input and the output radius of curvature of the surfaces.

to a minimum length distance D = 1.68 cm; while for α = 0.6, the minimum value for f' is 0.37 which corresponds to a minimum length distance D = 0.71 cm.

Next, for the second two designs, we selected f' = 1.75 cm (D = 3.384), k = 0.5, r_o = 2 (50% of central lob width), for α = 1 and α = 0.6, as shown in Figure 5 and Figure 6. Note that when k decreases by a factor of 2, then r_i decreases by the same factor. However, the minimum values for f' are changed to 1.58 (D = 3.01 cm) and 1.33 (D = 2.57 cm), for α = 1 and 0.6, respectively. This may explain why f' = 1.75 cm was selected in our design. Table 1 summarizes these results and includes others the designs parameters such as for α = 1 and k = 2 for both 50% and 95% central lobe widths.

As an illustration of the derivation of the mathematical expressions of the lens surfaces $y_i(r_i)$ and $y_o(r_o)$, we present the derivation of the design of Figure 3. We choose f' = 1.35 cm, k = 1, r_o = 2, α = 1. Substituting f' in EQ. (7) and using both (3) and (11), we can obtain the numerical values for the surface slopes dy_i/dr and dy_o/dr_o. The mathematical expressions for these slopes are achieved using a polynomial curve-fitting routine to yield:

$$\frac{dy_i}{dr_i} = (4.3210)r_i^4 - (8.9124)r_i^3 + (6.0735)r_i^2 - (1.3532)r_i + 0.0572 \quad (12a)$$

$$\frac{dy_o}{dr_o} = (0.0935)r_o^4 - (0.1875)r_o^3 + (0.3307)r_o^2 - (0.1099)r_0 - 0.0073 \quad (12b)$$

Integrating the above expressions to provide

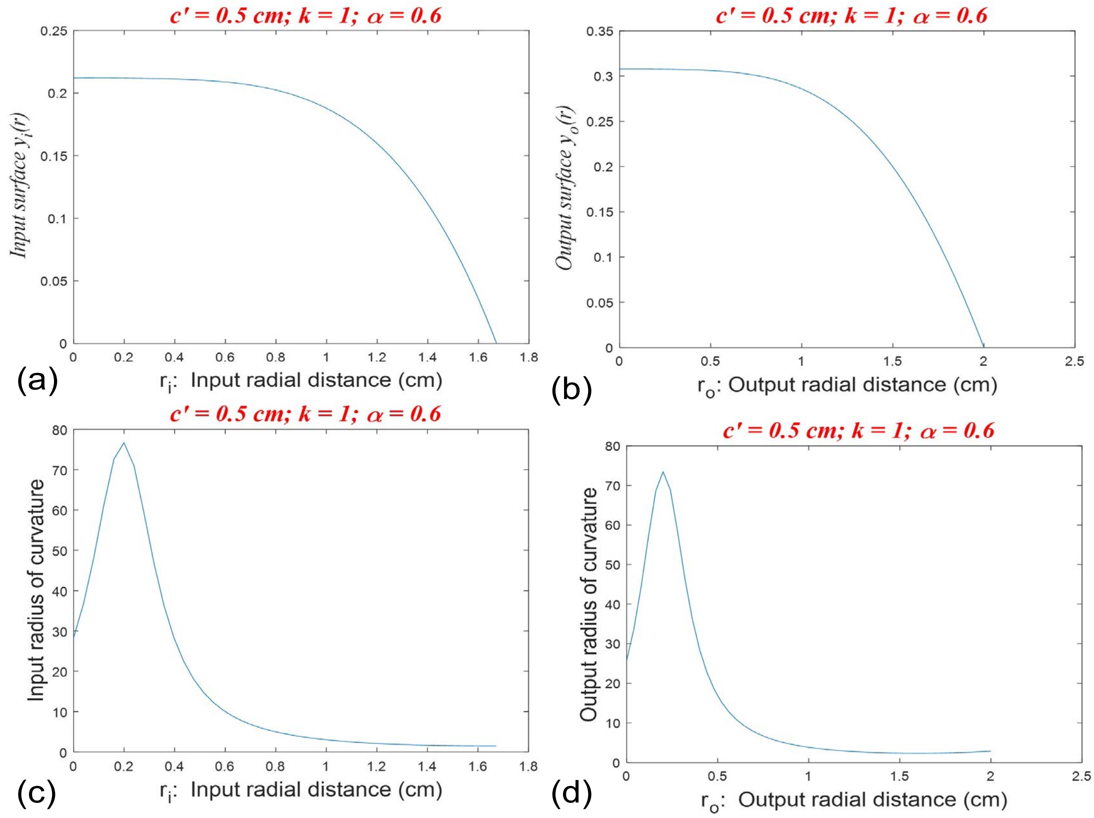

Figure 8: Two-element design for uniform to Bessel for c' = 0.5, D = 0.96, k = 1, α = 0.6: a) and b) The input and the output surfaces; c) and d) The input and the output radius of curvature of the surfaces.

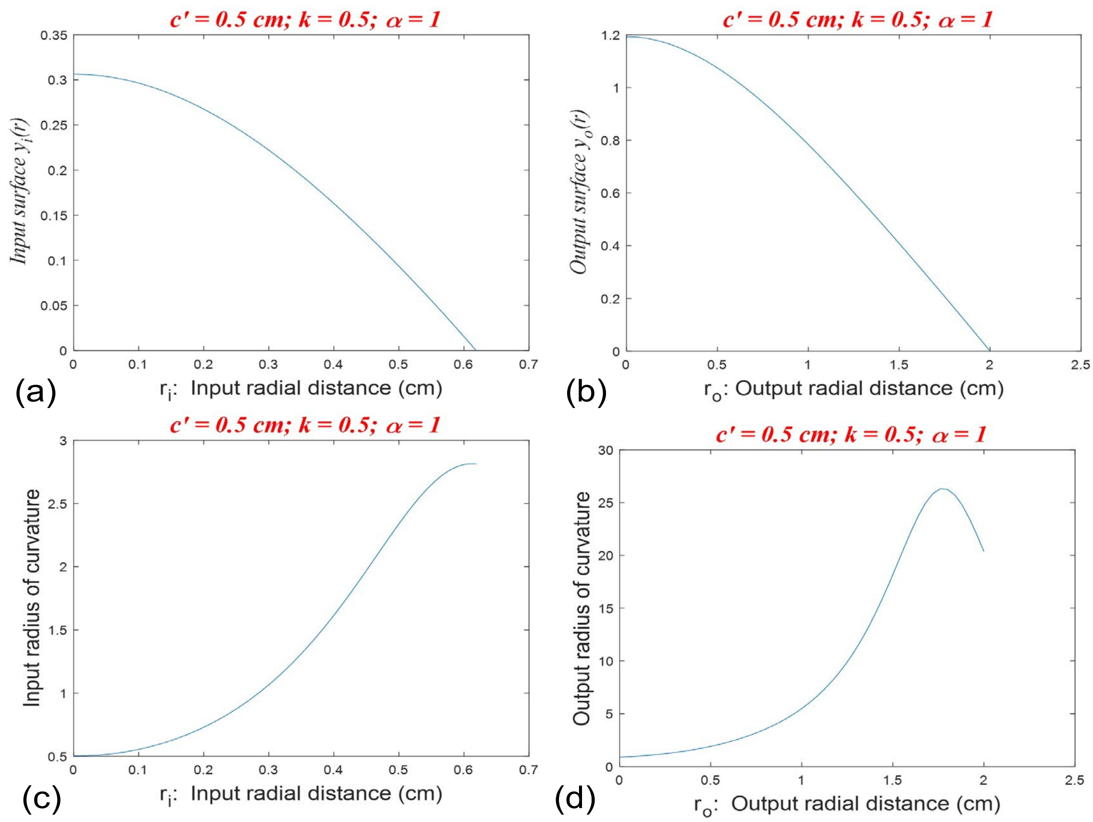

Figure 9: Two-element design for uniform to Bessel for c' = 0.5, D = 0.96, k = 0.5, α = 1: a) and b) The input and the output surfaces; c) and d) The input and the output radius of curvature of the surfaces.

approximate equations for the lens' surfaces as:

$$y_i(r_i) = (0.8642)r_i^5 - (2.2281)r_i^4 + (2.0245)r_i^3 - (0.6766)r_i^2 + (0.0572)r_i \quad (13a)$$

$$y_o(r_o) = (0.0187)r_o^5 - (0.0469)r_o^4 + (0.1102)r_o^3 - (0.0550)r_o^2 - (0.0073)r_0 \quad (13b)$$

Two-element design

Similarly, for the two-element design, Figure 7, Figure 8, Figure 9 and Figure 10 illustrate the four designs to demonstrate the effect of parameters k and α. Note that, in contrary to Eq. (5), Eq. (9) does not suffer from any singularity for the chosen glass with n = 1.5172.

For the first two designs, we have chosen the same design parameters as for the one-element design, i.e., k = 1, r_o = 2 (50% of central lob width), for α = 1 (r_i = 1.24) and α = 0.6 (r_i = 1.68). However, we have selected a much smaller system length c' = 0.5 cm (D = 0.96 cm) compared to D = 2.61 cm. These designs are shown in Figure 7 and Figure 8. In these figures, when α decreases, then the input radial distance r_i increases for a fixed k.

Next, to demonstrate the effect of k on the design, we have reduced k to 0.5 and kept the same parameters as the previous two designs. These designs are shown in Figure 9 and Figure 10. Since there is no singularity issue in Eq. (9), then we can still use the same value of c' = 0.5 cm (D = 0.96). The effect of k is restricted to only changing the value of r_i. Again, the two-element design achieved smaller system length than the one-element one. However, accurate alignment and precise separation between the two lenses are essential to achieve the desired output profile.

Conclusion

One- and two-element lens designs for transforming laser beam profiles are considered in this work using optical refracting system. The lens-design is applied to transform circular-uniform beam to be focused on the central beam lobe of a Bessel beam profile. In laser material processing, extending the DOF is very important. The minimal spreading of the central lobe of the Bessel beam during wave propagation makes extending the DOF possible. A procedure for obtaining the mathematical expressions for the two aspheric surfaces is outlined. An example was provided to illustrate the procedure. In addition, few important parameters such as the length of the system, the power ratio of the beams, and the first zero crossing of

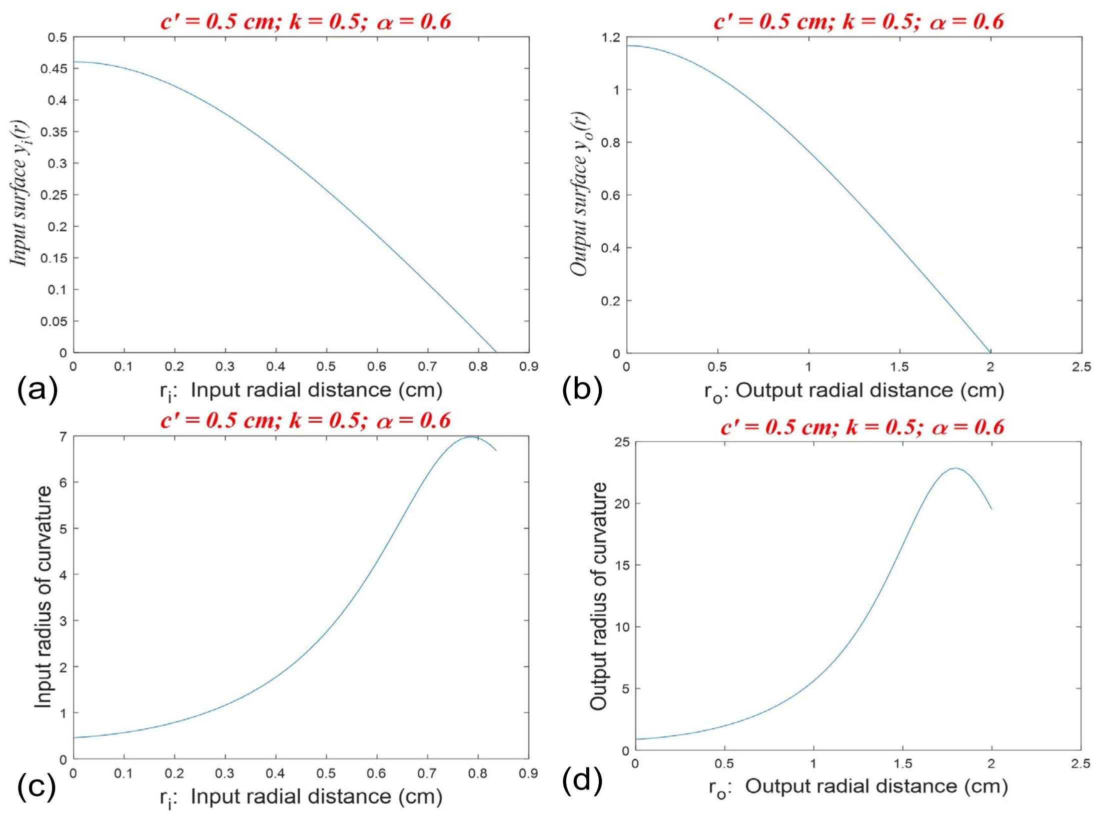

Figure 10: Two-element design for uniform to Bessel for c' = 0.5, D = 0.96, k = 0.5, α = 0.6: a) and b) The input and the output surfaces; c) and d) The input and the output radius of curvature of the surfaces.

the Bessel function, are discussed. The radii of curvature of the designed surfaces are high. This leads to smooth and gradual change on the surfaces for relative ease for machining and fabrication. It can be concluded that the two-element design will produce smaller system length than the one-element design.

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